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Modeling Hierarchical and Higher-Order Uncertainty in IT Systems Management Using Strong Intuitionistic Fuzzy Hypergraphs and Superhypergraphs

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ABSTRACT

Hypergraphs extend classical graphs by allowing hyperedges to connect more than two vertices, providing a natural framework for representing higher-order relationships in complex systems. SuperHyperGraphs further generalize this concept through iterated powerset constructions, enabling the modeling of hierarchical, set-valued, and multi-level interaction structures. At the same time, intuitionistic fuzzy theory offers an effective mechanism for handling uncertainty by simultaneously representing membership and non-membership information. A strong intuitionistic fuzzy graph constitutes a special class in which edge membership and non-membership degrees are directly induced by the corresponding extremal degrees of their incident vertices. In this paper, we extend the strong intuitionistic fuzzy paradigm from graphs to hypergraphs and SuperHyperGraphs. Specifically, we introduce the notions of strong intuitionistic fuzzy hypergraphs and strong intuitionistic fuzzy (n)-SuperHyperGraphs and investigate their fundamental properties. We establish generalization relationships with existing graph-based models and analyze several structural characteristics, including monotonicity, inheritance under sub-superhypergraph restrictions, and threshold-based crisp core representations. The proposed framework provides a unified approach for modeling hierarchical and higher-order uncertain relationships in complex systems. Illustrative applications in IT cost management and IT configuration management demonstrate its capability to support the representation and analysis of multi-level dependencies and uncertainty in engineering management environments.

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1. Introduction

Networks are fundamental mathematical models for representing relationships among objects. In classical graph theory, objects are represented by vertices and binary relationships are represented by edges. This framework is highly effective when the relevant interactions are pairwise. However, many real-world systems involve collective interactions among several entities at the same time. For example, a service process may depend jointly on several components, a project activity may require the simultaneous coordination of multiple teams, and an information system may involve dependencies among several configuration items. Such situations cannot always be represented adequately by ordinary graph edges.

Hypergraphs provide a natural extension of graphs by allowing each hyperedge to connect an arbitrary nonempty subset of vertices. Hence, hypergraphs are able to represent genuine higher-order relationships directly [1, 2]. Nevertheless, in many complex systems, relations are not only higher-order but also hierarchical, nested, or multi-level. To describe such structures, Smarandache introduced the notion of a SuperHyperGraph, in which iterated powerset constructions are used to generate higher-level vertices and edges [3]. In this setting, vertices may themselves be set-valued objects, and edges may describe incidence relations among these higher-level objects. Therefore, SuperHyperGraphs provide a flexible framework for modeling hierarchical networks and multi-layer incidence patterns. Recent studies have shown that SuperHyperGraphs are useful in both theoretical investigations and applied network modeling [4]. For a broader account of HyperGraphs and SuperHyperGraphs, see [5].

Graphs, hypergraphs, and SuperHyperGraphs are also important because they offer a transparent language for visualizing and analyzing complex systems. They have been used in artificial intelligence, network science, data mining, informatics, chemistry, physics, and related fields. When hierarchical organization is essential, SuperHyperGraphs can represent multi-level structural dependencies that are difficult to express by ordinary graphs or even by ordinary hypergraphs. This makes them suitable for the study of modern networked systems, including systems with nested resources, multi-component processes, and hierarchical decision units (e.g., [6]). Table 1 summarizes the main distinctions among graphs, hypergraphs, and n-SuperHyperGraphs.

In many applications, structural relationships are accompanied by uncertainty. For this reason, graph-theoretic models have often been combined with uncertainty formalisms such as fuzzy sets [8, 9], intuitionistic fuzzy sets [10], neutrosophic sets [11–13], uncertain sets [14, 15], and plithogenic sets [16, 17]. These combinations have led to many important network models, including fuzzy graphs, intuitionistic fuzzy graphs, neutrosophic graphs, plithogenic graphs, and their variants. Related concepts such as fuzzy graphs [9], neutrosophic graphs [18], neutrosophic soft graphs [19], plithogenic graphs [20], and hesitant fuzzy graphs [21] have also been studied in the literature. Moreover, fuzzy graphs, intuitionistic fuzzy graphs, and neutrosophic graphs have been extended in the directions of hypergraphs and SuperHyperGraphs [22–24].

This paper focuses on intuitionistic fuzzy graph structures. An intuitionistic fuzzy graph assigns membership and non-membership degrees to vertices and edges, subject to compatibility conditions determined by the incident vertices [25]. A strong intuitionistic fuzzy graph is a more restrictive and structurally tighter model in which the membership degree of an edge is exactly the minimum of the membership degrees of its endpoints, while the non-membership degree of the edge is exactly the maximum of the non-membership degrees of its endpoints [26]. This strong condition is meaningful in conservative decision modeling: the reliability of a relation is governed by its weakest incident component, while its uncertainty or non-membership is governed by its most uncertain incident component. As a reference, a comparison table of classical graphs, intuitionistic fuzzy graphs, and strong intuitionistic fuzzy graphs is presented in Table 2.

Although strong intuitionistic fuzzy graphs are useful for pairwise uncertain relations, they do not

Table 1
Key distinctions among graphs, hypergraphs, and n-SuperHyperGraphs.

Concept	*Notation*	*Edge family*	*Core extension principle*
Graph	$G = (V, E)$	$E \subseteq \binom{V}{2}$	Edges encode *pairwise* binary relations between vertices.
Hypergraph [7]	$H = (V, \mathcal{E})$	$\mathcal{E} \subseteq \mathrm{PWS}(V) \setminus \{\emptyset\}$	Hyperedges may connect any nonempty subset of vertices, thereby encoding *higher-order* interactions.
n-SuperHyperGraph [3]	$\mathrm{SupHG}^{(n)} = (V, E)$ (on a base set V_0)	$V \subseteq \mathrm{PWS}^n(V_0) \setminus \{\emptyset\}$, $E \subseteq \mathrm{PWS}(V) \setminus \{\emptyset\}$	Vertices may be set-valued objects in an n-fold powerset hierarchy, while edges are formed among these supervertices; this supports *nested* and *multi-level* incidence patterns.

Notation. $\mathrm{PWS}(X) = \{A \mid A \subseteq X\}$, $\binom{V}{2} = \{\{u, v\} \subseteq V \mid u \neq v\}$, and
$\mathrm{PWS}^0(X) = X,\ \mathrm{PWS}^{k+1}(X) = \mathrm{PWS}(\mathrm{PWS}^k(X))$.

directly represent higher-order or hierarchical interactions. Hypergraphs overcome the pairwise limitation, and SuperHyperGraphs further allow nested and multi-level structural objects. However, a systematic extension of strong intuitionistic fuzzy graphs to hypergraph and SuperHyperGraph settings has not yet been sufficiently developed. This motivates the present study.

The main purpose of this paper is to extend the notion of a strong intuitionistic fuzzy graph to higher-order and hierarchical network structures. More precisely, we introduce *strong intuitionistic fuzzy hypergraphs* and *strong intuitionistic fuzzy n-SuperHyperGraphs*. In the proposed models, the membership degree of each hyperedge or superedge is determined by the minimum membership degree of its incident vertices or supervertices, and the non-membership degree is determined by the maximum non-membership degree of its incident vertices or supervertices. This gives a direct generalization of the strong intuitionistic fuzzy graph principle to higher-order and hierarchical incidence structures.

The contributions of this paper can be summarized as follows:

1. We formulate strong intuitionistic fuzzy hypergraphs as a higher-order extension of strong intuitionistic fuzzy graphs.
2. We introduce strong intuitionistic fuzzy n-SuperHyperGraphs as a hierarchical extension based on n-fold powerset structures.
3. We prove that the proposed framework generalizes crisp hypergraphs, crisp n-SuperHyperGraphs, strong intuitionistic fuzzy graphs, and strong intuitionistic fuzzy hypergraphs.
4. We establish several fundamental properties, including monotonicity under inclusion, bounds induced by incident vertices, restriction to sub-superhypergraphs, and threshold-based crisp cores.
5. We provide illustrative applications to IT system management, especially IT cost management and IT configuration management.

Table 2
Classical graphs, intuitionistic fuzzy graphs, and strong intuitionistic fuzzy graphs.

Model	Vertex data	Edge data	Main feature
Classical graph	Crisp vertices $v \in V$	Crisp edges $uv \in E$	Represents ordinary pairwise relations.
Intuitionistic fuzzy graph	$(\sigma_A(v), \nu_A(v))$	$(\sigma_B(uv), \nu_B(uv))$, with $\sigma_B(uv) \leq \min\{\sigma_A(u), \sigma_A(v)\}$ and $\nu_B(uv) \geq \max\{\nu_A(u), \nu_A(v)\}$	Represents uncertain pairwise relations.
Strong intuitionistic fuzzy graph	$(\sigma_A(v), \nu_A(v))$	$\sigma_B(uv) = \min\{\sigma_A(u), \sigma_A(v)\}$, $\nu_B(uv) = \max\{\nu_A(u), \nu_A(v)\}$	Edge degrees are exactly induced by endpoint degrees.

The proposed framework is especially suitable for situations in which uncertain relations depend on several components simultaneously and where those components may themselves have internal structure. In IT system management, for instance, a cost-control process may involve cloud resources, security tools, vendor support, and organizational chargeback policies at the same time. Similarly, a configuration-management process may involve application configuration, database configuration, network rules, release automation, and audit evidence. Strong intuitionistic fuzzy SuperHyperGraphs provide a compact way to model such hierarchical uncertain dependencies.
This paper is theoretical in nature. Its scope is limited to a conceptual and mathematical investigation of the proposed structures. Computational experiments, simulations, algorithmic implementations, and empirical validation are left for future research.

2. Preliminaries

This section introduces the notation and foundational concepts used in the sequel. Unless explicitly stated otherwise, all graph-like structures considered in this paper are finite, undirected, and loopless. Parallel edges are allowed only when they are specified in the corresponding definition.

2.1 *SuperHyperGraphs*

In ordinary graph theory, a system of binary relations is modeled by a set of vertices together with a set of edges. Hypergraphs extend this representation by permitting one edge to be incident with any finite nonempty collection of vertices. In this way, hypergraphs provide a direct language for multiway or higher-order interactions [7, 27].
SuperHyperGraphs provide a further level of abstraction. Instead of taking vertices only as primitive objects, a SuperHyperGraph may use set-valued objects obtained from iterated powerset constructions over a fixed base set. Edges are then formed among such higher-level objects, allowing the representation of nested, hierarchical, and multi-level incidence relations [3]. Related hierarchical graph-like structures have appeared in several recent studies [28, 29]. For a more detailed discussion of SuperHyperGraphs and their variants, see the survey monograph [5].

Definition 2.1 (Iterated powerset)**.** [30] Let X be a set and let $k \in \mathbb{N}_0$. The k-fold iterated powerset of X is defined recursively by

$$\mathrm{PWS}^0(X) := X, \qquad \mathrm{PWS}^{k+1}(X) := \mathrm{PWS}\big(\mathrm{PWS}^k(X)\big).$$

Thus, $\mathrm{PWS}^1(X) = \mathrm{PWS}(X)$, $\mathrm{PWS}^2(X) = \mathrm{PWS}(\mathrm{PWS}(X))$, and so on.
Similarly, the nonempty iterated powerset construction is given by

$$(\mathrm{PWS}^*)^0(X) := X, \qquad (\mathrm{PWS}^*)^{k+1}(X) := \mathrm{PWS}^*\big((\mathrm{PWS}^*)^k(X)\big).$$

This version excludes the empty set at each positive level of iteration.

Definition 2.2 (Hypergraph)**.** [7, 31] A *hypergraph* is a pair $H = (V(H), E(H))$, where $V(H)$ is a nonempty finite set of vertices and

$$E(H) \subseteq \mathrm{PWS}^*(V(H)).$$

Each element of $E(H)$ is called a *hyperedge*. Hence a hyperedge is a nonempty subset of $V(H)$, and it may contain more than two vertices. Throughout this paper, both $V(H)$ and $E(H)$ are assumed to be finite.

Definition 2.3 (n-SuperHyperGraph)**.** [3] Let V_0 be a finite base set, and let $n \in \mathbb{N}_0$. An *n-SuperHyperGraph over V_0* is a triple

$$\mathrm{SupHG}^{(n)} = (V, E, \partial),$$

satisfying the following conditions:

- $V \subseteq \mathrm{PWS}^n(V_0)$ is a finite family of *n-supervertices*;
- E is a finite set whose elements are called *superedge identifiers*;
- $\partial : E \to \mathrm{PWS}^*(V)$ is an *incidence map*. For each $e \in E$, the value $\partial(e)$ is a nonempty finite subset of V.

The subset $\partial(e) \subseteq V$ is called the *incidence set* of e. It may also be called the *superincidence set*, since its elements are supervertices rather than necessarily primitive vertices.

Remark 2.4 (Common variants)**.** Several standard restrictions may be imposed on an n-SuperHyperGraph.

(i) The structure is called *simple* if the incidence map ∂ is injective. Equivalently, two distinct superedge identifiers cannot have the same incidence set.

(ii) The structure is called *k-uniform* if every superedge is incident with exactly k supervertices, that is,

$$|\partial(e)| = k \qquad \text{for all } e \in E.$$

(iii) If empty sets are to be excluded at every level of the hierarchy, one may require

$$V \subseteq (\mathrm{PWS}^*)^n(V_0).$$

This condition ensures that the supervertices are generated through the nonempty powerset construction at each tier.

Remark 2.5 (Subset presentation)**.** If parallel superedges are not needed, then each superedge identifier can be identified with its incidence set. Under this convention, an n-SuperHyperGraph may be written simply as a pair

$$(V, \mathcal{E}),$$

where

$$\mathcal{E} \subseteq \mathrm{PWS}^*(V).$$

This subset presentation is equivalent to Definition 2.3 in the simple case. Indeed, one may take $E := \mathcal{E}$ and define

$$\partial := \mathrm{id}_{\mathcal{E}},$$

so that every superedge is identified with the nonempty subset of supervertices to which it is incident.

Example 2.6 (A concrete 2-SuperHyperGraph)**.** Let the finite base set be

$$V_0 = \{a, b, c, d\}.$$

Then

$$\mathrm{PWS}^1(V_0) = \mathrm{PWS}(V_0), \qquad \mathrm{PWS}^2(V_0) = \mathrm{PWS}\big(\mathrm{PWS}(V_0)\big).$$

Thus, an element of $\mathrm{PWS}^2(V_0)$ is a set whose elements are subsets of V_0.
Define three 2-supervertices by

$$v_1 = \big\{\{a, b\}, \{c\}\big\}, \qquad v_2 = \big\{\{b, c\}, \{d\}\big\}, \qquad v_3 = \big\{\{a\}, \{c, d\}\big\}.$$

Since each v_i is a family of subsets of V_0, we have

$$v_1, v_2, v_3 \in \mathrm{PWS}^2(V_0).$$

Set

$$V = \{v_1, v_2, v_3\} \subseteq \mathrm{PWS}^2(V_0).$$

Let the set of superedge identifiers be

$$E = \{e_1, e_2, e_3\},$$

and define the incidence map

$$\partial : E \to \mathrm{PWS}^*(V)$$

by

$$\partial(e_1) = \{v_1, v_2\}, \qquad \partial(e_2) = \{v_2, v_3\}, \qquad \partial(e_3) = \{v_1, v_2, v_3\}.$$

Then

$$\mathrm{SupHG}^{(2)} = (V, E, \partial)$$

is a 2-SuperHyperGraph over V_0. The superedge e_1 connects v_1 and v_2, the superedge e_2 connects v_2 and v_3, and the superedge e_3 connects all three 2-supervertices.
A schematic representation is shown in Figure 1. Each large node represents a 2-supervertex, namely a set of subsets of the base set V_0. The dashed regions represent superedges determined by the incidence map ∂.
This example illustrates the key distinction between an ordinary hypergraph and a 2-SuperHyperGraph. In a hypergraph, vertices are elements of the base set. In the present 2-SuperHyperGraph, however, the vertices are higher-level objects:

$$v_i \in \mathrm{PWS}^2(V_0) = \mathrm{PWS}\big(\mathrm{PWS}(V_0)\big).$$

Therefore, each superedge relates not primitive elements of V_0, but families of subsets of V_0.

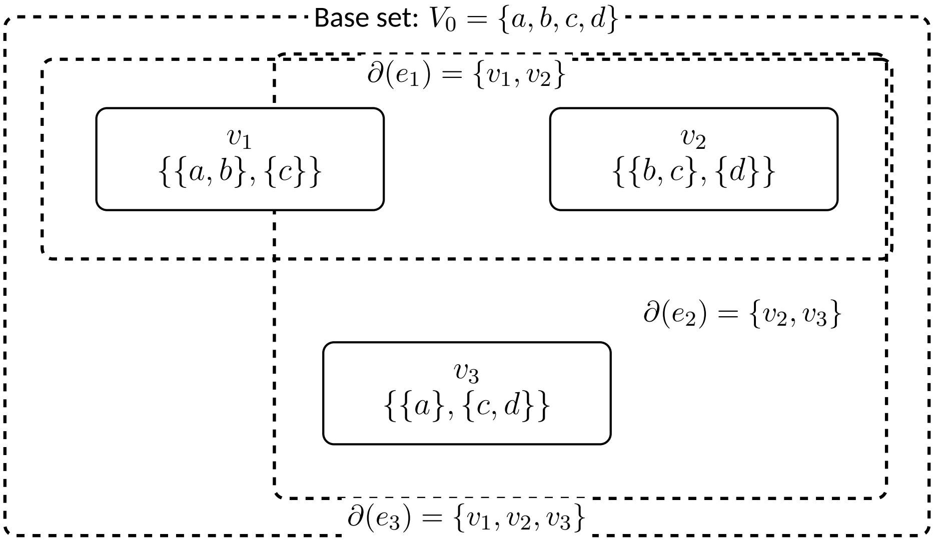

Fig. 1. A schematic 2-SuperHyperGraph over $V_0 = \{a, b, c, d\}$.

2.2 *Fuzzy n-SuperHyperGraphs*

Fuzzy set theory provides a standard mathematical framework for representing gradual membership by assigning to each element a value in the unit interval $[0, 1]$ [8]. When this idea is incorporated into graph-like structures, vertices and edges can be endowed with membership degrees, so that relationships are no longer treated merely as present or absent. In this way, fuzzy graphs and fuzzy hypergraphs provide useful models for graded connectivity, partial participation, and uncertain incidence relations [9, 24].

The same principle can be extended to SuperHyperGraphs. Since an n-SuperHyperGraph contains supervertices generated through an iterated powerset hierarchy, membership degrees may be assigned not only to ordinary objects but also to higher-level set-valued objects. Similarly, superedges may be assigned degrees that describe the strength, reliability, or validity of hierarchical relations among such supervertices. Thus, fuzzy n-SuperHyperGraphs offer a natural framework for modeling uncertain, graded, and hierarchical interactions (cf. [3, 32]).

Definition 2.7 (Fuzzy graph). [9, 33] A *fuzzy graph* is a triple

$$G = (V, \sigma_V, \sigma_E),$$

where V is a finite nonempty vertex set,

$$\sigma_V : V \to [0, 1]$$

is the vertex-membership function, and

$$\sigma_E : V \times V \to [0, 1]$$

is the edge-membership function. These maps are required to satisfy

$$\sigma_E(u, v) \leq \min\{\sigma_V(u), \sigma_V(v)\} \qquad (\forall\, u, v \in V).$$

Thus, the membership degree of an edge cannot exceed the membership degree of either of its incident vertices. For a crisp edge $uv := \{u, v\}$, we write

$$\sigma_E(uv) := \sigma_E(u, v).$$

The associated crisp graph has vertex set V and edge set

$$E^* := \{\, uv \mid \sigma_E(uv) > 0 \,\}.$$

Definition 2.8 (Fuzzy hypergraph)**.** [34] Let $H^* = (V, E, \partial)$ be a crisp hypergraph. A *fuzzy hypergraph* based on H^* is a sextuple

$$\mathcal{H} = (V, E, \partial; \sigma_V, \sigma_E, \eta),$$

where

$$\sigma_V : V \to [0,1], \qquad \sigma_E : E \to [0,1], \qquad \eta : V \times E \to [0,1].$$

Here σ_V, σ_E, and η denote the vertex-membership, hyperedge-membership, and incidence-membership functions, respectively. These functions satisfy the following conditions for every $v \in V$ and every $e \in E$:

$$\textbf{(support condition)} \quad v \in \partial(e) \quad \Longleftrightarrow \quad \eta(v,e) > 0, \tag{1}$$

$$\textbf{(incidence compatibility)} \quad \eta(v,e) \le \min\{\sigma_V(v), \sigma_E(e)\}, \tag{2}$$

$$\textbf{(hyperedge compatibility)} \quad \sigma_E(e) \le \min_{u \in \partial(e)} \sigma_V(u). \tag{3}$$

The support condition states that positive incidence membership occurs exactly for those vertex–hyperedge incidences that are present in the underlying crisp hypergraph. Consequently, the crisp hypergraph (V, E, ∂) can be recovered from (1).

Definition 2.9 (Fuzzy n-SuperHyperGraph)**.** (cf. [3, 35]) Let

$$\mathrm{SupHG}^{(n)} = (V, E)$$

be an n-SuperHyperGraph in the subset presentation, so that

$$E \subseteq \mathrm{PWS}^{*}(V).$$

A *fuzzy n-SuperHyperGraph* is a quadruple

$$\mathbb{G}^{(n)} = (V, E, \sigma_V, \sigma_E),$$

where

$$\sigma_V : V \to [0,1], \qquad \sigma_E : E \to [0,1].$$

The map σ_V assigns a membership degree to each supervertex, while σ_E assigns a membership degree to each superedge. These maps are required to satisfy the admissibility condition

$$\sigma_E(e) \le \min_{v \in e} \sigma_V(v) \qquad (\forall\, e \in E).$$

Hence, the degree assigned to a superedge is bounded above by the smallest membership degree among its incident supervertices.

2.3 *Intuitionistic fuzzy graphs and strong intuitionistic fuzzy graphs*

An intuitionistic fuzzy graph enriches an ordinary graph by assigning two degrees to each vertex and each edge: a membership degree and a non-membership degree. These degrees are required to satisfy natural compatibility conditions with respect to incidence, so that the uncertainty attached to an edge is controlled by the uncertainty of its endpoints [36]. A strong intuitionistic fuzzy graph is a special form of an intuitionistic fuzzy graph in which the edge degrees are not chosen independently; rather, they are determined exactly by the extremal membership and non-membership degrees of the incident vertices [26].

Definition 2.10 (Intuitionistic fuzzy graph and strong intuitionistic fuzzy graph)**.** [26] Let

$$G^* = (V, E)$$

be a finite simple undirected graph, where

$$E \subseteq \{\{u, v\} \mid u, v \in V,\ u \neq v\}.$$

An *intuitionistic fuzzy graph* on G^* is a pair

$$G = (A, B),$$

where $A = (\sigma_A, \nu_A)$ is an intuitionistic fuzzy set on the vertex set V, and $B = (\sigma_B, \nu_B)$ is an intuitionistic fuzzy set on the edge set E. More precisely,

$$\sigma_A : V \to [0, 1], \qquad \nu_A : V \to [0, 1],$$

and

$$0 \leq \sigma_A(x) + \nu_A(x) \leq 1 \qquad (\forall\, x \in V).$$

Here $\sigma_A(x)$ and $\nu_A(x)$ denote the membership and non-membership degrees of the vertex x, respectively.
Similarly,

$$\sigma_B : E \to [0, 1], \qquad \nu_B : E \to [0, 1],$$

and

$$0 \leq \sigma_B(uv) + \nu_B(uv) \leq 1 \qquad (\forall\, uv \in E).$$

The edge degrees are required to be compatible with the endpoint degrees. That is, for every edge $uv \in E$,

$$\sigma_B(uv) \leq \min\{\sigma_A(u), \sigma_A(v)\}, \qquad \nu_B(uv) \geq \max\{\nu_A(u), \nu_A(v)\}.$$

Thus, the membership degree of an edge cannot exceed the smaller membership degree of its two endpoints, while the non-membership degree of the edge cannot be smaller than the larger non-membership degree of its endpoints.
The intuitionistic fuzzy graph $G = (A, B)$ is called a *strong intuitionistic fuzzy graph* if the above inequalities are attained as equalities for every edge. Namely, for all $uv \in E$,

$$\sigma_B(uv) = \min\{\sigma_A(u), \sigma_A(v)\}, \qquad \nu_B(uv) = \max\{\nu_A(u), \nu_A(v)\}.$$

In this case, each edge degree is completely induced by the degrees of its incident vertices.

3. Main Results

This section develops the principal constructions of the paper. We first introduce strong intuitionistic fuzzy hypergraphs as higher-order analogues of strong intuitionistic fuzzy graphs. We then extend the same idea to n-SuperHyperGraphs and establish several basic structural properties.

3.1 Strong intuitionistic fuzzy hypergraphs

A strong intuitionistic fuzzy hypergraph is obtained by assigning membership and non-membership degrees to vertices and hyperedges in such a way that every hyperedge degree is determined exactly by the extremal degrees of its incident vertices.

Definition 3.1 (Intuitionistic fuzzy hypergraph)**.** Let

$$H = (V, E)$$

be a finite hypergraph, where $V \neq \varnothing$ and

$$E \subseteq \mathrm{PWS}^{*}(V).$$

An *intuitionistic fuzzy hypergraph* on H is a tuple

$$\mathbb{H} = \big(V, E; \sigma_V, \nu_V, \sigma_E, \nu_E\big),$$

where (σ_V, ν_V) is an intuitionistic fuzzy set on V, and (σ_E, ν_E) is an intuitionistic fuzzy set on E. In addition, the following compatibility conditions are required for every hyperedge $e \in E$:

$$\sigma_E(e) \leq \min_{v \in e} \sigma_V(v), \qquad \nu_E(e) \geq \max_{v \in e} \nu_V(v). \tag{4}$$

Thus, the membership degree of a hyperedge cannot exceed the smallest membership degree of its incident vertices, while its non-membership degree cannot be smaller than the largest non-membership degree among those vertices.

Definition 3.2 (Strong intuitionistic fuzzy hypergraph)**.** Let

$$\mathbb{H} = \big(V, E; \sigma_V, \nu_V, \sigma_E, \nu_E\big)$$

be an intuitionistic fuzzy hypergraph. It is called a *strong intuitionistic fuzzy hypergraph* if, for every $e \in E$,

$$\sigma_E(e) = \min_{v \in e} \sigma_V(v), \qquad \nu_E(e) = \max_{v \in e} \nu_V(v). \tag{5}$$

Hence, in the strong case, the degrees assigned to a hyperedge are not independent data; they are induced exactly by the extremal membership and non-membership degrees of the vertices incident with that hyperedge.
Since (σ_E, ν_E) is required to be an intuitionistic fuzzy set on E, we also have

$$\min_{v \in e} \sigma_V(v) + \max_{v \in e} \nu_V(v) \leq 1 \qquad (\forall\, e \in E).$$

Example 3.3 (A strong intuitionistic fuzzy hypergraph)**.** Let

$$V = \{v_1, v_2, v_3, v_4\}$$

be a finite vertex set, and define the hyperedge family

$$E = \{e_1, e_2, e_3\},$$

where

$$e_1 = \{v_1, v_2, v_3\}, \qquad e_2 = \{v_2, v_4\}, \qquad e_3 = \{v_1, v_3, v_4\}.$$

Then

$$H = (V, E)$$

is a finite hypergraph, since every hyperedge is a nonempty subset of V.
Assign membership and non-membership degrees to the vertices as follows:

Vertex	σ_V	ν_V
v_1	0.80	0.10
v_2	0.65	0.20
v_3	0.72	0.12
v_4	0.55	0.30

For each $v \in V$, we have

$$0 \leq \sigma_V(v) + \nu_V(v) \leq 1.$$

Hence (σ_V, ν_V) is an intuitionistic fuzzy set on V.
Now define the hyperedge degrees by the strong rule

$$\sigma_E(e) = \min_{v \in e} \sigma_V(v), \qquad \nu_E(e) = \max_{v \in e} \nu_V(v).$$

Then we obtain

Hyperedge	Incident vertices	σ_E	ν_E
e_1	$\{v_1, v_2, v_3\}$	$\min\{0.80, 0.65, 0.72\} = 0.65$	$\max\{0.10, 0.20, 0.12\} = 0.20$
e_2	$\{v_2, v_4\}$	$\min\{0.65, 0.55\} = 0.55$	$\max\{0.20, 0.30\} = 0.30$
e_3	$\{v_1, v_3, v_4\}$	$\min\{0.80, 0.72, 0.55\} = 0.55$	$\max\{0.10, 0.12, 0.30\} = 0.30$

Moreover,

$$0.65 + 0.20 = 0.85 \leq 1, \qquad 0.55 + 0.30 = 0.85 \leq 1.$$

Therefore, (σ_E, ν_E) is an intuitionistic fuzzy set on E.
Thus

$$\mathbb{H} = \big(V, E; \sigma_V, \nu_V, \sigma_E, \nu_E\big)$$

is a strong intuitionistic fuzzy hypergraph. In particular, the membership degree of each hyperedge is determined by the weakest membership degree among its incident vertices, while the non-membership degree is determined by the largest non-membership degree among its incident vertices.
A schematic representation is given in Figure 2. The dashed regions represent hyperedges, and each vertex label shows the pair (σ_V, ν_V).

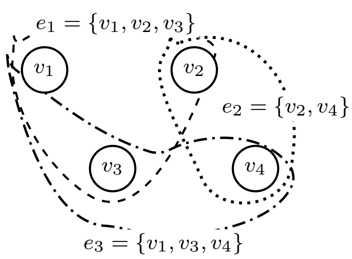

Fig. 2. A schematic strong intuitionistic fuzzy hypergraph.

Theorem 3.4 (Strong intuitionistic fuzzy hypergraphs generalize hypergraphs and strong intuitionistic fuzzy graphs)**.** *The following statements hold.*

(i) *Every crisp hypergraph is a special case of a strong intuitionistic fuzzy hypergraph.*

(ii) *Every strong intuitionistic fuzzy graph can be regarded as a special case of a strong intuitionistic fuzzy hypergraph.*

Proof. **(i)** Let $H = (V, E)$ be a crisp hypergraph. Define

$$\sigma_V(v) := 1, \qquad \nu_V(v) := 0 \qquad (v \in V),$$

and

$$\sigma_E(e) := 1, \qquad \nu_E(e) := 0 \qquad (e \in E).$$

Then (σ_V, ν_V) is an intuitionistic fuzzy set on V, and (σ_E, ν_E) is an intuitionistic fuzzy set on E. Moreover, for each hyperedge $e \in E$,

$$\min_{v \in e} \sigma_V(v) = 1 = \sigma_E(e), \qquad \max_{v \in e} \nu_V(v) = 0 = \nu_E(e).$$

Therefore the strong equalities in (5) are satisfied. Consequently,

$$\mathbb{H} := \big(V, E; \sigma_V, \nu_V, \sigma_E, \nu_E\big)$$

is a strong intuitionistic fuzzy hypergraph whose underlying crisp structure is exactly H.
(ii) Let $G^* = (V, E)$ be a finite simple undirected graph, and let $G = (A, B)$ be a strong intuitionistic fuzzy graph on G^*. Write

$$A = (\sigma_A, \nu_A), \qquad B = (\sigma_B, \nu_B).$$

By the strong condition, for every edge $uv \in E$,

$$\sigma_B(uv) = \min\{\sigma_A(u), \sigma_A(v)\}, \qquad \nu_B(uv) = \max\{\nu_A(u), \nu_A(v)\}.$$

Now regard G^* as a 2-uniform hypergraph by identifying each graph edge uv with the two-element hyperedge $\{u, v\}$. Thus

$$E \subseteq \mathrm{PWS}^*(V).$$

Define

$$\sigma_V := \sigma_A, \qquad \nu_V := \nu_A \quad \text{on } V,$$

and

$$\sigma_E := \sigma_B, \qquad \nu_E := \nu_B \quad \text{on } E.$$

Then (σ_V, ν_V) is an intuitionistic fuzzy set on V, and (σ_E, ν_E) is an intuitionistic fuzzy set on E. For every hyperedge $e = \{u, v\} \in E$, we have

$$\min_{x \in e} \sigma_V(x) = \min\{\sigma_A(u), \sigma_A(v)\} = \sigma_B(uv) = \sigma_E(e),$$

and similarly,

$$\max_{x \in e} \nu_V(x) = \max\{\nu_A(u), \nu_A(v)\} = \nu_B(uv) = \nu_E(e).$$

Hence (5) holds for every hyperedge. Therefore the given strong intuitionistic fuzzy graph is recovered as a 2-uniform strong intuitionistic fuzzy hypergraph. $\square$

3.2 Strong intuitionistic fuzzy superhypergraphs

A strong intuitionistic fuzzy superhypergraph extends the preceding construction from hypergraphs to SuperHyperGraphs. In this setting, membership and non-membership degrees are assigned to supervertices and superedges, and each superedge degree is induced exactly by the extremal degrees of its incident supervertices.

Definition 3.5 (Strong intuitionistic fuzzy n-SuperHyperGraph in subset presentation)**.** Let V_0 be a finite base set, and let $n \in \mathbb{N}_0$. Consider an n-SuperHyperGraph in the subset presentation,

$$\mathrm{SupHG}^{(n)} = (V, \mathcal{E}),$$

where

$$V \subseteq \mathrm{PWS}^n(V_0)$$

is a finite set of n-supervertices, and

$$\mathcal{E} \subseteq \mathrm{PWS}^*(V)$$

is a finite family of nonempty subsets of V. The elements of $\mathcal{E}$ are called superedges.
An *intuitionistic fuzzy n-SuperHyperGraph* on $(V, \mathcal{E})$ is a tuple

$$\mathbb{S} = \big(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\big),$$

where (σ_V, ν_V) is an intuitionistic fuzzy set on V, and $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$. These maps are required to satisfy, for every superedge $e \in \mathcal{E}$,

$$\sigma_{\mathcal{E}}(e) \le \min_{v \in e} \sigma_V(v), \qquad \nu_{\mathcal{E}}(e) \ge \max_{v \in e} \nu_V(v). \tag{6}$$

The intuitionistic fuzzy n-SuperHyperGraph $\mathbb{S}$ is called a *strong intuitionistic fuzzy n-SuperHyperGraph* if the above inequalities are equalities for every superedge. That is,

$$\sigma_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v), \qquad \nu_{\mathcal{E}}(e) = \max_{v \in e} \nu_V(v) \qquad (\forall\, e \in \mathcal{E}). \tag{7}$$

Thus, in the strong case, the membership and non-membership degrees of each superedge are completely determined by the incident supervertices.

Theorem 3.6 (Strong intuitionistic fuzzy SuperHyperGraphs as a common generalization)**.** *The following statements hold.*

(i) *Every crisp n-SuperHyperGraph is a special case of a strong intuitionistic fuzzy n-SuperHyperGraph.*

(ii) *Every strong intuitionistic fuzzy hypergraph is a special case of a strong intuitionistic fuzzy 0-SuperHyperGraph.*

(iii) *Every strong intuitionistic fuzzy graph is a special case of a strong intuitionistic fuzzy 0-SuperHyperGraph.*

Proof. **(i)** Let

$$\mathrm{SupHG}^{(n)} = (V, \mathcal{E})$$

be a crisp n-SuperHyperGraph in the subset presentation. Define

$$\sigma_V(v) := 1, \qquad \nu_V(v) := 0 \qquad (v \in V),$$

and

$$\sigma_{\mathcal{E}}(e) := 1, \qquad \nu_{\mathcal{E}}(e) := 0 \qquad (e \in \mathcal{E}).$$

Then (σ_V, ν_V) is an intuitionistic fuzzy set on V, and $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$. For every $e \in \mathcal{E}$,

$$\min_{v\in e} \sigma_V(v) = 1 = \sigma_{\mathcal{E}}(e), \qquad \max_{v\in e} \nu_V(v) = 0 = \nu_{\mathcal{E}}(e).$$

Hence the strong equalities in (7) hold. Therefore

$$\bigl(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\bigr)$$

is a strong intuitionistic fuzzy n-SuperHyperGraph whose underlying crisp structure is precisely $(V, \mathcal{E})$.
(ii) Let

$$\mathbb{H} = \bigl(V, E; \sigma_V^H, \nu_V^H, \sigma_E^H, \nu_E^H\bigr)$$

be a strong intuitionistic fuzzy hypergraph. Put

$$V_0 := V, \qquad n := 0.$$

Since

$$\mathrm{PWS}^0(V_0) = V_0,$$

the vertex set V is a subset of $\mathrm{PWS}^0(V_0)$. Moreover, because $E \subseteq \mathrm{PWS}^*(V)$, the hyperedge family E is a valid superedge family for a 0-SuperHyperGraph. Thus (V, E) may be viewed as a 0-SuperHyperGraph in the subset presentation.
Set

$$\mathcal{E} := E, \qquad \sigma_V := \sigma_V^H, \qquad \nu_V := \nu_V^H,$$

and

$$\sigma_{\mathcal{E}} := \sigma_E^H, \qquad \nu_{\mathcal{E}} := \nu_E^H.$$

Then

$$\mathbb{S} = \bigl(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\bigr)$$

is an intuitionistic fuzzy 0-SuperHyperGraph.
Since $\mathbb{H}$ is strong, for every $e \in E = \mathcal{E}$,

$$\sigma_{\mathcal{E}}(e) = \sigma_E^H(e) = \min_{v\in e} \sigma_V^H(v) = \min_{v\in e} \sigma_V(v),$$

and

$$\nu_{\mathcal{E}}(e) = \nu_E^H(e) = \max_{v\in e} \nu_V^H(v) = \max_{v\in e} \nu_V(v).$$

Therefore $\mathbb{S}$ satisfies the strong equalities (7). Hence every strong intuitionistic fuzzy hypergraph is realized as a strong intuitionistic fuzzy 0-SuperHyperGraph.
(iii) Let $G = (A, B)$ be a strong intuitionistic fuzzy graph on a finite simple graph $G^* = (V, E)$. By Theorem 3.4(ii), G can be regarded as a strong intuitionistic fuzzy hypergraph by identifying each graph edge with a two-element hyperedge. By part **(ii)**, this strong intuitionistic fuzzy hypergraph is then a special case of a strong intuitionistic fuzzy 0-SuperHyperGraph. Hence every strong intuitionistic fuzzy graph is also a special case of a strong intuitionistic fuzzy 0-SuperHyperGraph. □

Theorem 3.7 (Superedge feasibility). *Let*

$$\mathbb{S} = \bigl(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\bigr)$$

be a strong intuitionistic fuzzy n-SuperHyperGraph. Then, for every superedge $e \in \mathcal{E}$,

$$0 \le \sigma_{\mathcal{E}}(e) + \nu_{\mathcal{E}}(e) \le 1.$$

Consequently, $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$.

Proof. The lower bound follows from the fact that all membership and non-membership degrees lie in $[0, 1]$.
For the upper bound, fix $e \in \mathcal{E}$. Choose $w \in e$ such that

$$\nu_V(w) = \max_{v \in e} \nu_V(v).$$

Since (σ_V, ν_V) is an intuitionistic fuzzy set on V, we have

$$\sigma_V(w) + \nu_V(w) \le 1.$$

Moreover,

$$\min_{v \in e} \sigma_V(v) \le \sigma_V(w).$$

Using the strong equalities, we obtain

$$\sigma_{\mathcal{E}}(e) + \nu_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v) + \max_{v \in e} \nu_V(v) \le \sigma_V(w) + \nu_V(w) \le 1.$$

Therefore

$$0 \le \sigma_{\mathcal{E}}(e) + \nu_{\mathcal{E}}(e) \le 1.$$

Thus $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$. □

Theorem 3.8 (Monotonicity with respect to superedge inclusion). *Let $e, f \in \mathcal{E}$ satisfy $e \subseteq f$. Then*

$$\sigma_{\mathcal{E}}(f) \le \sigma_{\mathcal{E}}(e), \qquad \nu_{\mathcal{E}}(f) \ge \nu_{\mathcal{E}}(e).$$

Proof. Since $e \subseteq f$, taking the minimum over the larger set f cannot increase the value. Hence

$$\min_{v \in f} \sigma_V(v) \le \min_{v \in e} \sigma_V(v).$$

By (7),

$$\sigma_{\mathcal{E}}(f) = \min_{v \in f} \sigma_V(v) \le \min_{v \in e} \sigma_V(v) = \sigma_{\mathcal{E}}(e).$$

Similarly, taking the maximum over the larger set f cannot decrease the value, so

$$\max_{v \in f} \nu_V(v) \ge \max_{v \in e} \nu_V(v).$$

Again using (7), we get

$$\nu_{\mathcal{E}}(f) = \max_{v \in f} \nu_V(v) \ge \max_{v \in e} \nu_V(v) = \nu_{\mathcal{E}}(e).$$

This proves both inequalities. □

Theorem 3.9 (Bounds induced by incident supervertices). *Let $e \in \mathcal{E}$ and let $v \in e$. Then*

$$\sigma_{\mathcal{E}}(e) \le \sigma_V(v), \qquad \nu_{\mathcal{E}}(e) \ge \nu_V(v).$$

Proof. Since v is one of the supervertices incident with e, we have

$$\min_{u \in e} \sigma_V(u) \le \sigma_V(v), \qquad \max_{u \in e} \nu_V(u) \ge \nu_V(v).$$

Applying the strong equalities in (7) gives

$$\sigma_{\mathcal{E}}(e) = \min_{u \in e} \sigma_V(u) \le \sigma_V(v),$$

and

$$\nu_{\mathcal{E}}(e) = \max_{u \in e} \nu_V(u) \ge \nu_V(v).$$

□

Theorem 3.10 (Attainment of the extremal values). *Let $e \in \mathcal{E}$. Choose $v_{\min}, v_{\max} \in e$ such that*

$$\sigma_V(v_{\min}) = \min_{v\in e} \sigma_V(v), \qquad \nu_V(v_{\max}) = \max_{v\in e} \nu_V(v).$$

Then

$$\sigma_{\mathcal{E}}(e) = \sigma_V(v_{\min}), \qquad \nu_{\mathcal{E}}(e) = \nu_V(v_{\max}).$$

In particular, the inequalities in Theorem 3.9 are sharp for at least one incident supervertex in each component.

Proof. Because every superedge is finite and nonempty, the minimum of σ_V and the maximum of ν_V over e are attained. By the choice of $v_{\min}$ and $v_{\max}$, and by (7), we have

$$\sigma_{\mathcal{E}}(e) = \min_{v\in e} \sigma_V(v) = \sigma_V(v_{\min}),$$

and

$$\nu_{\mathcal{E}}(e) = \max_{v\in e} \nu_V(v) = \nu_V(v_{\max}).$$

Therefore the stated equalities follow. □

Theorem 3.11 (Restriction to an induced sub-superhypergraph). *Let $V' \subseteq V$, and define*

$$\mathcal{E}' := \{\, e \in \mathcal{E} \mid e \subseteq V' \,\}.$$

Let

$$\sigma_{V'} := \sigma_V|_{V'}, \qquad \nu_{V'} := \nu_V|_{V'},$$

and

$$\sigma_{\mathcal{E}'} := \sigma_{\mathcal{E}}|_{\mathcal{E}'}, \qquad \nu_{\mathcal{E}'} := \nu_{\mathcal{E}}|_{\mathcal{E}'}.$$

Then

$$\mathbb{S}' = \left(V', \mathcal{E}'; \sigma_{V'}, \nu_{V'}, \sigma_{\mathcal{E}'}, \nu_{\mathcal{E}'}\right)$$

is a strong intuitionistic fuzzy n-SuperHyperGraph on the induced pair $(V', \mathcal{E}')$.

Proof. Since $V' \subseteq V$ and $\mathcal{E}' \subseteq \mathcal{E}$, all four restricted maps are well defined. Let $e \in \mathcal{E}'$. By definition of $\mathcal{E}'$, we have $e \subseteq V'$. Hence the restrictions $\sigma_{V'}$ and $\nu_{V'}$ agree with σ_V and ν_V on every element of e. Therefore

$$\min_{v\in e} \sigma_{V'}(v) = \min_{v\in e} \sigma_V(v),$$

and

$$\max_{v\in e} \nu_{V'}(v) = \max_{v\in e} \nu_V(v).$$

Using the strong equalities for $\mathbb{S}$, we obtain

$$\sigma_{\mathcal{E}'}(e) = \sigma_{\mathcal{E}}(e) = \min_{v\in e} \sigma_V(v) = \min_{v\in e} \sigma_{V'}(v),$$

and

$$\nu_{\mathcal{E}'}(e) = \nu_{\mathcal{E}}(e) = \max_{v\in e} \nu_V(v) = \max_{v\in e} \nu_{V'}(v).$$

Thus $\mathbb{S}'$ satisfies the strong intuitionistic fuzzy conditions on every superedge $e \in \mathcal{E}'$. Hence $\mathbb{S}'$ is a strong intuitionistic fuzzy n-SuperHyperGraph. □

Theorem 3.12 (Threshold-induced crisp core). *Let* $\alpha, \beta \in [0, 1]$. *Define*

$$V_{\alpha,\beta} := \{ v \in V \mid \sigma_V(v) \geq \alpha,\ \nu_V(v) \leq \beta \},$$

and

$$\mathcal{E}_{\alpha,\beta} := \{ e \in \mathcal{E} \mid \sigma_{\mathcal{E}}(e) \geq \alpha,\ \nu_{\mathcal{E}}(e) \leq \beta \}.$$

Then

$$(V_{\alpha,\beta}, \mathcal{E}_{\alpha,\beta})$$

is a crisp n*-SuperHyperGraph in the subset presentation. Equivalently,*

$$\mathcal{E}_{\alpha,\beta} \subseteq \mathrm{PWS}^*(V_{\alpha,\beta}).$$

Proof. Let $e \in \mathcal{E}_{\alpha,\beta}$. By definition,

$$\sigma_{\mathcal{E}}(e) \geq \alpha, \qquad \nu_{\mathcal{E}}(e) \leq \beta.$$

Now take any $v \in e$. By Theorem 3.9,

$$\sigma_V(v) \geq \sigma_{\mathcal{E}}(e), \qquad \nu_V(v) \leq \nu_{\mathcal{E}}(e).$$

Combining these inequalities yields

$$\sigma_V(v) \geq \alpha, \qquad \nu_V(v) \leq \beta.$$

Therefore $v \in V_{\alpha,\beta}$. Since this holds for every $v \in e$, we obtain

$$e \subseteq V_{\alpha,\beta}.$$

Because e is nonempty, it follows that

$$e \in \mathrm{PWS}^*(V_{\alpha,\beta}).$$

Thus

$$\mathcal{E}_{\alpha,\beta} \subseteq \mathrm{PWS}^*(V_{\alpha,\beta}),$$

and consequently $(V_{\alpha,\beta}, \mathcal{E}_{\alpha,\beta})$ is a crisp n-SuperHyperGraph in the subset presentation. □

4. Illustrative IT Management Examples of Strong Intuitionistic Fuzzy SuperHyperGraphs

In this section, we present two illustrative examples showing how strong intuitionistic fuzzy SuperHyperGraphs can be used to model hierarchical and uncertain relationships in IT management. The first example concerns IT cost management (cf.[37]), while the second concerns IT configuration management.

Example 4.1 (IT cost management as a strong intuitionistic fuzzy SuperHyperGraph). Consider an IT cost management environment in which several primitive cost-related items must be monitored jointly. Let the base set be

$$V_0 = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\},$$

where

$c_1 =$ cloud compute cost, $c_2 =$ database service cost, $c_3 =$ software license cost,
$c_4 =$ monitoring-tool cost, $c_5 =$ vendor support cost, $c_6 =$ security-tool cost,
$c_7 =$ departmental chargeback policy.

We construct a 1-SuperHyperGraph in the subset presentation. The set of supervertices is

$$V = \{v_1, v_2, v_3, v_4, v_5\} \subseteq \mathrm{PWS}(V_0),$$

where

$$\begin{aligned} v_1 &= \{c_1, c_2\}, \\ v_2 &= \{c_3\}, \\ v_3 &= \{c_4, c_6\}, \\ v_4 &= \{c_5\}, \\ v_5 &= \{c_7\}. \end{aligned}$$

Here v_1 represents the cloud infrastructure cost block, v_2 represents the software licensing block, v_3 represents the monitoring–security tool cost block, v_4 represents the vendor support block, and v_5 represents the organizational chargeback policy block.
Let the family of superedges be

$$\mathcal{E} = \{e_1, e_2, e_3, e_4\} \subseteq \mathrm{PWS}^*(V),$$

where

$$\begin{aligned} e_1 &= \{v_1, v_3, v_4\}, \\ e_2 &= \{v_1, v_2, v_5\}, \\ e_3 &= \{v_1, v_3, v_4, v_5\}, \\ e_4 &= \{v_2, v_4, v_5\}. \end{aligned}$$

The superedge e_1 represents the monthly cloud-cost control cycle involving cloud resources, monitoring/security tools, and vendor support. The superedge e_2 represents a licensing and chargeback adjustment process. The superedge e_3 represents a budget-overrun remediation process involving infrastructure, security/monitoring, vendor support, and chargeback rules. The superedge e_4 represents a vendor and license renegotiation process.
Assign intuitionistic fuzzy degrees to the supervertices as follows:

Supervertex	Meaning	σ_V	ν_V
v_1	cloud infrastructure cost block	0.82	0.10
v_2	software licensing block	0.76	0.15
v_3	monitoring–security tool cost block	0.70	0.18
v_4	vendor support block	0.74	0.16
v_5	chargeback policy block	0.68	0.22

For each $v \in V$, we have

$$0 \leq \sigma_V(v) + \nu_V(v) \leq 1.$$

Thus, (σ_V, ν_V) is an intuitionistic fuzzy set on V.
Now define the superedge degrees by the strong intuitionistic fuzzy rule

$$\sigma_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v), \qquad \nu_{\mathcal{E}}(e) = \max_{v \in e} \nu_V(v).$$

Then we obtain

Superedge	Incident supervertices	$\sigma_{\mathcal{E}}$	$\nu_{\mathcal{E}}$
e_1	$\{v_1, v_3, v_4\}$	$\min\{0.82, 0.70, 0.74\}$ $= 0.70$	$\max\{0.10, 0.18, 0.16\}$ $= 0.18$
e_2	$\{v_1, v_2, v_5\}$	$\min\{0.82, 0.76, 0.68\}$ $= 0.68$	$\max\{0.10, 0.15, 0.22\}$ $= 0.22$
e_3	$\{v_1, v_3, v_4, v_5\}$	$\min\{0.82, 0.70, 0.74, 0.68\}$ $= 0.68$	$\max\{0.10, 0.18, 0.16, 0.22\}$ $= 0.22$
e_4	$\{v_2, v_4, v_5\}$	$\min\{0.76, 0.74, 0.68\}$ $= 0.68$	$\max\{0.15, 0.16, 0.22\}$ $= 0.22$

For each $e \in \mathcal{E}$, we have

$$0 \le \sigma_{\mathcal{E}}(e) + \nu_{\mathcal{E}}(e) \le 1.$$

Therefore, $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$.
Moreover, for every $e \in \mathcal{E}$, the equalities

$$\sigma_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v), \qquad \nu_{\mathcal{E}}(e) = \max_{v \in e} \nu_V(v)$$

hold. Hence

$$\mathbb{S}_{\text{cost}} = \big(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\big)$$

is a strong intuitionistic fuzzy 1-SuperHyperGraph.
In this model, the membership degree $\sigma_V(v)$ represents the reliability, controllability, or budgetary stability of a cost block, whereas the non-membership degree $\nu_V(v)$ represents the degree of cost uncertainty, cost leakage, or budget-overrun risk. The strong superedge rule means that the reliability of a multi-block cost-management process is governed by its weakest participating cost block, while its risk is governed by the riskiest participating block. This is natural in IT cost management because a cost-control workflow can fail or become unreliable if even one essential cost component is poorly controlled.

Example 4.2 (IT configuration management as a strong intuitionistic fuzzy SuperHyperGraph)**.** Consider an IT configuration management process in which application, database, network, deployment, and audit-related configuration items must be controlled jointly. Let the base set be

$$V_0 = \{d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8\},$$

where

$d_1 =$ web application configuration, $d_2 =$ application server configuration,
$d_3 =$ database schema configuration, $d_4 =$ firewall rule configuration,
$d_5 =$ load-balancer configuration, $d_6 =$ CI/CD pipeline configuration,
$d_7 =$ CMDB record, $d_8 =$ monitoring alert configuration.

We construct a 1-SuperHyperGraph by grouping primitive configuration items into higher-level configuration blocks. Let

$$V = \{u_1, u_2, u_3, u_4, u_5\} \subseteq \mathrm{PWS}(V_0),$$

where

$$\begin{aligned} u_1 &= \{d_1, d_2\}, \\ u_2 &= \{d_3\}, \\ u_3 &= \{d_4, d_5\}, \\ u_4 &= \{d_6\}, \\ u_5 &= \{d_7, d_8\}. \end{aligned}$$

The supervertex u_1 represents the application runtime configuration block, u_2 represents the database configuration block, u_3 represents the network exposure configuration block, u_4 represents the release automation configuration block, and u_5 represents the configuration evidence and observability block.
Let the family of superedges be

$$\mathcal{E} = \{f_1, f_2, f_3, f_4\} \subseteq \mathrm{PWS}^*(V),$$

where

$$\begin{aligned} f_1 &= \{u_1, u_2, u_3\}, \\ f_2 &= \{u_1, u_4, u_5\}, \\ f_3 &= \{u_2, u_3, u_5\}, \\ f_4 &= \{u_1, u_2, u_3, u_4, u_5\}. \end{aligned}$$

The superedge f_1 represents a baseline dependency configuration involving application, database, and network settings. The superedge f_2 represents a controlled release update involving runtime configuration, CI/CD configuration, and CMDB/monitoring evidence. The superedge f_3 represents a configuration audit involving database, network, and CMDB-related evidence. The superedge f_4 represents an emergency change package involving all configuration blocks.
Assign intuitionistic fuzzy degrees to the supervertices as follows:

Supervertex	Meaning	σ_V	ν_V
u_1	application runtime configuration block	0.88	0.08
u_2	database configuration block	0.80	0.12
u_3	network exposure configuration block	0.72	0.18
u_4	release automation configuration block	0.78	0.15
u_5	configuration evidence and observability block	0.75	0.16

For every $u \in V$, we have

$$0 \le \sigma_V(u) + \nu_V(u) \le 1.$$

Therefore, (σ_V, ν_V) is an intuitionistic fuzzy set on V.
Define the superedge degrees by

$$\sigma_{\mathcal{E}}(f) = \min_{u \in f} \sigma_V(u), \qquad \nu_{\mathcal{E}}(f) = \max_{u \in f} \nu_V(u).$$

Then the following values are obtained:

Superedge	Incident supervertices	$\sigma_{\mathcal{E}}$	$\nu_{\mathcal{E}}$
f_1	$\{u_1, u_2, u_3\}$	$\min\{0.88, 0.80, 0.72\} = 0.72$	$\max\{0.08, 0.12, 0.18\} = 0.18$
f_2	$\{u_1, u_4, u_5\}$	$\min\{0.88, 0.78, 0.75\} = 0.75$	$\max\{0.08, 0.15, 0.16\} = 0.16$
f_3	$\{u_2, u_3, u_5\}$	$\min\{0.80, 0.72, 0.75\} = 0.72$	$\max\{0.12, 0.18, 0.16\} = 0.18$
f_4	$\{u_1, u_2, u_3, u_4, u_5\}$	$\min\{0.88, 0.80, 0.72, 0.78, 0.75\} = 0.72$	$\max\{0.08, 0.12, 0.18, 0.15, 0.16\} = 0.18$

For every $f \in \mathcal{E}$, we have

$$0 \le \sigma_{\mathcal{E}}(f) + \nu_{\mathcal{E}}(f) \le 1.$$

Thus, $(\sigma_{\mathcal{E}}, \nu_{\mathcal{E}})$ is an intuitionistic fuzzy set on $\mathcal{E}$.
Furthermore, the strong equalities

$$\sigma_{\mathcal{E}}(f) = \min_{u \in f} \sigma_V(u), \qquad \nu_{\mathcal{E}}(f) = \max_{u \in f} \nu_V(u)$$

hold for every $f \in \mathcal{E}$. Hence

$$\mathbb{S}_{\text{config}} = \big(V, \mathcal{E}; \sigma_V, \nu_V, \sigma_{\mathcal{E}}, \nu_{\mathcal{E}}\big)$$

is a strong intuitionistic fuzzy 1-SuperHyperGraph.

In this example, the membership degree represents the reliability, correctness, or audit readiness of a configuration block, while the non-membership degree represents configuration drift, inconsistency risk, or lack of verification. The strong superedge rule reflects a conservative IT configuration management principle: a configuration workflow is only as reliable as its weakest participating configuration block, and its uncertainty is determined by the most uncertain participating block. This is especially meaningful for emergency changes, baseline updates, and audit-driven configuration reviews, where an unverified or inconsistent component can compromise the validity of the entire configuration process.

5. Discussion

This section clarifies the position of the proposed framework within the family of intuitionistic fuzzy graph-based models. Table 3 compares strong intuitionistic fuzzy graphs, strong intuitionistic fuzzy hypergraphs, and strong intuitionistic fuzzy superhypergraphs from the viewpoint of their underlying structures, admissible edge objects, and induced degree conditions. Table 4 further distinguishes a general intuitionistic fuzzy superhypergraph from its strong version.

Table 3

Comparison among strong intuitionistic fuzzy graphs, strong intuitionistic fuzzy hypergraphs, and strong intuitionistic fuzzy superhypergraphs.

Aspect	**Strong intuitionistic fuzzy graph**	**Strong intuitionistic fuzzy hypergraph**	**Strong intuitionistic fuzzy superhypergraph**
Basic structure	A graph $G = (V, E)$, in which every edge connects exactly two vertices.	A hypergraph $H = (V, E)$, in which a hyperedge may connect any nonempty subset of vertices.	An n-SuperHyperGraph $(V, \mathcal{E})$, where supervertices may be higher-level set-valued objects.
Vertex-type objects	Ordinary vertices $v \in V$.	Ordinary vertices $v \in V$.	Supervertices satisfying $v \in V \subseteq \mathrm{PWS}^n(V_0)$.
Edge-type objects	Ordinary edges $uv \in E$.	Hyperedges $e \in E \subseteq \mathrm{PWS}^*(V)$.	Superedges $e \in \mathcal{E} \subseteq \mathrm{PWS}^*(V)$.
Membership rule	The membership of an edge is given by $\sigma_B(uv) = \min\{\sigma_A(u), \sigma_A(v)\}$.	The membership of a hyperedge is given by $\sigma_E(e) = \min_{v \in e} \sigma_V(v)$.	The membership of a superedge is given by $\sigma_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v)$.
Non-membership rule	The non-membership of an edge is given by $\nu_B(uv) = \max\{\nu_A(u), \nu_A(v)\}$.	The non-membership of a hyperedge is given by $\nu_E(e) = \max_{v \in e} \nu_V(v)$.	The non-membership of a superedge is given by $\nu_{\mathcal{E}}(e) = \max_{v \in e} \nu_V(v)$.
Modeling purpose	Describes uncertain pairwise relations.	Describes uncertain higher-order relations among several vertices.	Describes uncertain relations that are both higher-order and hierarchical.

The first comparison shows that the proposed model follows a natural extension path:

$$\text{graph} \longrightarrow \text{hypergraph} \longrightarrow \text{SuperHyperGraph}.$$

At each stage, the same strong intuitionistic fuzzy principle is preserved. The membership degree of an edge-type object is determined by the weakest membership degree among its incident objects,

Table 4
Comparison between intuitionistic fuzzy superhypergraphs and strong intuitionistic fuzzy superhypergraphs.

Aspect	**Intuitionistic fuzzy superhypergraph**	**Strong intuitionistic fuzzy superhypergraph**
Underlying structure	Built on an n-SuperHyperGraph $(V, \mathcal{E})$.	Built on the same type of n-SuperHyperGraph $(V, \mathcal{E})$.
Supervertex information	Each supervertex $v \in V$ is assigned membership and non-membership degrees $\sigma_V(v)$ and $\nu_V(v)$.	Each supervertex $v \in V$ is assigned the same type of degrees $\sigma_V(v)$ and $\nu_V(v)$.
Superedge information	Each superedge $e \in \mathcal{E}$ has assigned degrees $\sigma_{\mathcal{E}}(e)$ and $\nu_{\mathcal{E}}(e)$, subject to compatibility bounds.	Each superedge $e \in \mathcal{E}$ has degrees determined exactly by the extremal degrees of its incident supervertices.
Membership condition	The superedge membership is bounded above by the smallest incident supervertex membership: $$\sigma_{\mathcal{E}}(e) \leq \min_{v \in e} \sigma_V(v).$$	The superedge membership coincides with the smallest incident supervertex membership: $$\sigma_{\mathcal{E}}(e) = \min_{v \in e} \sigma_V(v).$$
Non-membership condition	The superedge non-membership is bounded below by the largest incident supervertex non-membership: $$\nu_{\mathcal{E}}(e) \geq \max_{v \in e} \nu_V(v).$$	The superedge non-membership coincides with the largest incident supervertex non-membership: $$\nu_{\mathcal{E}}(e) = \max_{v \in e} \nu_V(v).$$
Interpretation	Superedge degrees are only constrained by the incident supervertices and may contain additional modeling freedom.	Superedge degrees are fully induced by the incident supervertices, leaving no independent choice at the superedge level.
Structural character	More flexible, since the superedge degrees may vary within admissible bounds.	More rigid, but also more canonical, because the superedge degrees are fixed by the incidence structure.

while the non-membership degree is determined by the strongest non-membership degree among them. Therefore, the proposed strong intuitionistic fuzzy superhypergraph model can be regarded as a hierarchical and higher-order extension of strong intuitionistic fuzzy graphs.

The second comparison emphasizes the difference between the general and strong versions. In a general intuitionistic fuzzy superhypergraph, the degrees of a superedge are only bounded by the degrees of its incident supervertices. This allows greater modeling flexibility. In the strong version, however, the superedge degrees are exactly determined by the incident supervertices. This makes the model more restrictive, but it also gives a clearer structural interpretation. In particular, the strong condition is appropriate for conservative decision-making situations in which a multi-component relation is considered reliable only to the extent that all of its participating components are reliable.

6. Conclusion

In this paper, we extended the notion of a strong intuitionistic fuzzy graph by employing hypergraphs and superhypergraphs. More precisely, we introduced *strong intuitionistic fuzzy hypergraphs* and *strong intuitionistic fuzzy superhypergraphs*, in which the membership and non-membership degrees of each hyperedge or superedge are determined by the extrema of the degrees of its incident vertices or supervertices. We established several fundamental results, including generalization theorems, monotonicity under inclusion, restriction to sub-superhypergraphs, and threshold-based crisp cores.
The main findings of this study show that the proposed structures provide a natural common extension of crisp hypergraphs, strong intuitionistic fuzzy graphs, and strong intuitionistic fuzzy hypergraphs. In particular, the strong condition gives a clear interpretation: the membership degree of a higher-order or hierarchical relation is governed by the weakest participating component, whereas its non-membership degree is governed by the most uncertain or least reliable component. The illustrative examples suggest that this framework can represent higher-order and hierarchical uncertain relations in practical decision-making settings, especially in IT cost management and IT configuration management.
This study also has some limitations. Since the work is mainly theoretical, the proposed models have not yet been evaluated through large-scale datasets, computational experiments, or empirical case studies. Moreover, algorithmic issues such as efficient construction, optimization, ranking, clustering, and decision-making procedures for strong intuitionistic fuzzy superhypergraphs remain open.
Future research may extend the present framework by incorporating richer uncertainty formalisms, such as neutrosophic graphs and plithogenic graphs. It would also be useful to develop computational algorithms, compare the proposed model with existing fuzzy and intuitionistic fuzzy network models, and examine further applications in complex service systems, engineering systems, information systems, and multi-criteria decision-making environments. Moreover, the concepts introduced in this paper may be further generalized by using related higher-order and hierarchical network structures, such as Recursive HyperGraphs, Meta-HyperGraphs, Recursive SuperHyperGraphs, and Hierarchical SuperHyperGraphs.

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Conflicts of Interest
The authors declare no conflicts of interest.

Declaration of Generative AI and AI-Assisted Technologies in the Manuscript Preparation Process
During the preparation of this work, the authors used AI-assisted tools ChatGPT-4 to enhance readability and improve language structure. These tools were not used to generate scientific content, analyses, or conclusions. After using these tools, the authors reviewed and edited the content as needed and take full responsibility for the final manuscript. No generative AI tools were used to create or modify the study figures or artwork.

References

[1] Feng, Y., You, H., Zhang, Z., Ji, R., & Gao, Y. (2019). Hypergraph neural networks. *Proceedings of the AAAI conference on artificial intelligence*, *33*(01), 3558–3565. https://arxiv.org/abs/1809.09401

[2] Cai, D., Song, M., Sun, C., Zhang, B., Hong, S., & Li, H. (2022). Hypergraph structure learning for hypergraph neural networks. *IJCAI*, 1923–1929.

[3] Smarandache, F. (2020). *Extension of hypergraph to n-superhypergraph and to plithogenic n-superhypergraph, and extension of hyperalgebra to n-ary (classical-/neutro-/anti-) hyperalgebra*. Infinite Study. https://fs.unm.edu/nss8/index.php/111/article/view/198

[4] Fujita, T., & Smarandache, F. (2026a). *Representing higher-order networks: A survey of graph-based frameworks*. Infinite Study.

[5] Fujita, T., & Smarandache, F. (2026b). *Hypergraph and superhypergraph theory with applications*. Neutrosophic Science International Association (NSIA) Publishing House. https://fs.unm.edu/HyperGraphSuperHyperGraphTheory1.pdf

[6] Berrocal Villegas, S. M., Montalvo Fritas, W., Berrocal Villegas, C. R., Flores Fuentes Rivera, M. Y., Espejo Rivera, R., Bautista Puma, L. D., & Macazana Fernández, D. M. (2025). Using plithogenic n-superhypergraphs to assess the degree of relationship between information skills and digital competencies. *Neutrosophic Sets and Systems*, *84*(1), 41. https://doi.org/10.5281/zenodo.15641273

[7] Bretto, A. (2013). Hypergraph theory. *An introduction. Mathematical Engineering. Cham: Springer*, *1*. https://doi.org/10.1007/978-3-319-00080-0

[8] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[9] Rosenfeld, A. (1975). Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes* (pp. 77–95). Elsevier. https://doi.org/10.1016/B978-0-12-775260-0.50008-6

[10] Atanassov, K. T. (2020). Circular intuitionistic fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, *39*(5), 5981–5986. https://doi.org/10.3233/JIFS-189072

[11] Broumi, S., Talea, M., Bakali, A., & Smarandache, F. (2016a). Single valued neutrosophic graphs. *Journal of New theory*, (10), 86–101. https://doi.org/10.5281/zenodo.50940

[12] Ghods, M., Rostami, Z., Fujita, T., & Smarandache, F. (2026). *Topological indices on single-valued neutrosophic graphs*. Neutrosophic Science International Association (NSIA) Publishing House. https://fs.unm.edu/NSS/IntroductionToTopological4.pdf

[13] Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). *Single valued neutrosophic sets*. Infinite study. https://fs.unm.edu/SingleValuedNeutrosophicSets.pdf

[14] Fujita, T., & Smarandache, F. (2026c). *A comprehensive survey of set-theoretic concepts related to fuzzy, neutrosophic, and uncertain sets*. Infinite Study. https://philpapers.org/archive/SMAACS.pdf

[15] Fujita, T., & Smarandache, F. (2026d). *Survey of fuzzy and uncertain logical operators*. Infinite Study. https://philarchive.org/archive/FUJSOF

[16] Sultana, F., Gulistan, M., Ali, M., Yaqoob, N., Khan, M., Rashid, T., & Ahmed, T. (2023). A study of plithogenic graphs: Applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, *14*(10), 13139–13159. https://doi.org/10.1007/s12652-022-03772-6

[17] Smarandache, F. (2018). *Plithogenic set, an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-revisited*. Infinite study. https://fs.unm.edu/NSS/PlithogenicSetAnExtensionOfCrisp.pdf

[18] Broumi, S., Talea, M., Bakali, A., & Smarandache, F. (2016b). Interval valued neutrosophic graphs. *Critical Review, XII, 2016*, 5–33. https://fs.unm.edu/IntervalValuedNeutrosophicGraphs-CR12.pdf

[19] Hussain, S., Hussain, J., Rosyida, I., & Broumi, S. (2022). Quadripartitioned neutrosophic soft graphs. In *Handbook of research on advances and applications of fuzzy sets and logic* (pp. 771–795). IGI Global. https://doi.org/10.4018/978-1-7998-7979-4.ch034

[20] Singh, P. K. (2022). Intuitionistic plithogenic graph and it's-cut for knowledge processing tasks. *Neutrosophic Sets and Systems*, *49*(1), 5. https://doi.org/10.5281/zenodo.6426373
[21] AbuHijleh, E. A. (2023). Complex hesitant fuzzy graph. *Fuzzy Information and Engineering*, *15*(2), 149–161. https://doi.org/10.26599/FIE.2023.9270010
[22] Dhanya, P. M., Sreekumar, A., Jathavedan, M., & Ramkumar, P. B. (2018). Algebra of morphological dilation on intuitionistic fuzzy hypergraphs. *International journal of scientific research in science, engineering and technology*, *4*, 300–308. https://api.semanticscholar.org/CorpusID:53656713
[23] Shalini, F. (2023). Trigonometric similarity measures of pythagorean neutrosophic hypersoft sets. *Neutrosophic Systems with Applications*. https://api.semanticscholar.org/CorpusID:261671130
[24] Mordeson, J. N., & Nair, P. S. (2012). *Fuzzy graphs and fuzzy hypergraphs* (Vol. 46). Physica. https://doi.org/10.1007/978-3-7908-1854-3
[25] Akram, M., Davvaz, B., & Feng, F. (2013). Intuitionistic fuzzy soft k-algebras. *Mathematics in Computer Science*, *7*, 353–365. https://doi.org/10.1007/s11786-013-0158-5
[26] Akram, M., & Davvaz, B. (2012). Strong intuitionistic fuzzy graphs. *Filomat*, *26*(1), 177–196. https://doi.org/10.2298/FIL1201177A
[27] Ausiello, G., & Laura, L. (2017). Directed hypergraphs: Introduction and fundamental algorithms-a survey. *Theoretical Computer Science*, *658*, 293–306.
[28] Hamidi, M., & Taghinezhad, M. (2023). *Application of superhypergraphs-based domination number in real world*. Infinite Study. https://doi.org/jmmr.2023.21203.1415
[29] Mogro, E. J., Molina, J. R., Canas, G. J. S., & Soria, P. H. (2024). Tree tobacco extract (*Nicotiana glauca*) as a plithogenic bioinsecticide alternative for controlling fruit fly (*Drosophila immigrans*) using n-superhypergraphs. *Neutrosophic Sets and Systems*, *74*, 57–65. https://fs.unm.edu/NSS/6TreeTobaccoExtract.pdf
[30] Smarandache, F. (2023). *Superhyperfunction, superhyperstructure, neutrosophic superhyperfunction and neutrosophic superhyperstructure: Current understanding and future directions*. Infinite Study. https://doi.org/10.61356/j.nswa.2023.115
[31] Berge, C. (1984). *Hypergraphs: Combinatorics of finite sets* (Vol. 45). Elsevier.
[32] Alqahtani, M. (2025). Intuitionistic fuzzy quasi-supergraph integration for social network decision making. *International Journal of Analysis and Applications*, *23*, 137–137.
[33] Akram, M., & Luqman, A. (2020). *Fuzzy hypergraphs and related extensions*. Springer. https://doi.org/10.1007/978-981-15-2403-5
[34] Bershtein, L. S., & Bozhenyuk, A. V. (2009). Fuzzy graphs and fuzzy hypergraphs. In *Encyclopedia of artificial intelligence* (pp. 704–709). IGI Global. https://doi.org/10.4018/978-1-59904-849-9.ch105
[35] Hamidi, M., Smarandache, F., & Taghinezhad, M. (2023). *Decision making based on valued fuzzy superhypergraphs*. Infinite Study. https://doi.org/10.32604/cmes.2023.030284
[36] Sahoo, S., & Pal, M. (2017). Product of intuitionistic fuzzy graphs and degree. *Journal of Intelligent & Fuzzy Systems*, *32*(1), 1059–1067. https://doi.org/10.3233/JIFS-16348
[37] Riedinger, C., & Huber, M. (2023). An expert view on challenges in managing it costs in the digital age. *IADIS IS 2023 Proceedings*. https://www.is-conf.org/is-2023-file/