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Application of MCDM Methods in Optimizing Measurement Accuracy for Geodetic Monitoring of Engineering Structure Displacements

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ABSTRACT

The purpose of this study is to develop and evaluate a decision-making framework for selecting the optimal measurement accuracy of geodetic instruments used in engineering structure displacement monitoring. The study investigates whether integrating multiple Multi-Criteria Decision-Making (MCDM) methods with different preference structures can provide reliable and robust support for this complex engineering geodesy problem under an unfavourable geodetic control network configuration. To address this question, the proposed framework combines the SWING weighting method with four MCDM methods, namely Proximity Indexed Value (PIV), Preference Analysis for Reference Ideal Solution (PARIS), Measurement of Alternatives and Ranking according to COMpromise Solution (MARCOS), and COMplex PROportional ASsessment (COPRAS). Six total station alternatives were evaluated using criterion functions representing measurement precision and reliability characteristics under four preference structures represented by ordered 8-tuples of criterion weights. The framework was validated through comparative analysis using TOPSIS as the benchmark method, while ranking robustness and consistency were further examined through a leave-one-out sensitivity analysis. The results demonstrate that all four MCDM methods successfully addressed the considered optimization problem and consistently identified the same two best-performing alternatives. The comparative analysis revealed a high degree of agreement between PIV, PARIS, COPRAS, and the TOPSIS benchmark, whereas MARCOS exhibited slightly greater ranking variability while preserving the identification of the optimal solutions. Furthermore, the sensitivity analysis showed that ranking stability was influenced more strongly by the adopted preference structure than by the particular MCDM method employed. These findings are significant because they establish a comprehensive and reproducible framework for optimizing measurement accuracy in engineering geodesy, demonstrate the applicability of four MCDM methods not previously employed for this optimization problem, and provide broader methodological insight into the importance of combining multiple preference structures, comparative analysis, and sensitivity analysis when evaluating MCDM performance.

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1. Introduction

Extensively documented in prior literature, classical geodetic optimization techniques have long been utilized for network design purposes. Conventionally, this optimization framework is divided into four distinct phases: Zero-Order Design (ZOD) – addresses the datum issue; First-Order Design (FOD) – focuses on the network configuration; Second-Order Design (SOD) – concerns the observations' weights issue; Third-Order Design (THOD) – deals with the optimal enhancement of an established geodetic network. Nevertheless, this categorization remains somewhat arbitrary. Its primary limitation stems from treating these optimization phases as isolated problems, whereas they frequently overlap in practice. For example, variations in the SOD design matrix inherently impact the FOD, etc.

Rather than relying on these traditional optimization methodologies, this study implements a Multi-Criteria Decision-Making (MCDM) framework. This represents a fundamentally different paradigm for special-purpose geodetic networks. It resolves the optimization challenge by simultaneously evaluating several competing and conflicting criteria to establish an optimal measurement accuracy in a geodetic control network.

1.1 MCDM Methods in Engineering Geodesy and Beyond: A Review

While MCDM approaches have gained substantial traction across numerous research domains, including recent advancements in environmental studies, photogrammetry, and land consolidation, their integration into engineering geodesy remains scarce. Namely, there are only a few papers dealing with the application of MCDM methods in the optimization of geodetic micro-networks. The adequate examples of such papers are some recent studies.

For example, in the study written by Kobryń [1], the author proposes the PROcessing TEchnique of Ratings for Ranking of Alternatives (PROTERRA) and applies the Analytic Hierarchy Process (AHP) [2], the Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) [3] and the TOPSIS [4] methods in the optimization of design of a geodetic network.

On the other hand, the paper conducted by Anđić and Đurović [5] deals with the use of the Multi-Criteria Optimization and Compromise Ranking/Solution (VIKOR; in Serbian: Višekriterijumsko KOMpromisno Rangiranje) [6], one of the most commonly used methods in various fields. The subject of optimization in that study was the design of a geodetic micro-network with only measured distances, established for the purpose of detecting deviations of the projected from the marked structure pillars' positions. The competitive alternative solutions were represented by different geodetic network designs with varying the number of reference points and distribution of measurements, but involving only one geodetic instrument, i.e. total station.

Another study worth mentioning was written by Anđić [7]. The paper demonstrates a successful application of the Multi-Attributive Border Approximation Area Comparison (MABAC) [8], the Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) [9], the COmbined COmpromise SOlution (CO-COSO) [10] and the Range Of Value (ROV) [11] methods in engineering geodesy. The subject of the study was a tower geodetic network, represented with varying design, in which both horizontal directions and distances were measured using one total station.

The most recent study dealing with application of MCDM methods in the optimization of geodetic networks was conducted by Anđić [12]. This time, a dam geodetic network, with measured both horizontal directions and distances, was optimized. Alternatives were established by varying the design and using a specific total station as a measuring instrument for each of the alternatives. This study showed that the Multiple Criteria Ranking by Alternative Trace (MCRAT) [13], the Ranking the Alternatives by Perimeter Similarity (RAPS) [13], the Ranking by Alternatives Median Similarity (RAMS) [14] and the Ranking by Alternatives using the Trace to Median Index (RATMI) [14] methods can serve as a useful tool when it comes to optimizing geodetic works performed with the aim of determining the displacement of an engineering structure between two consecutive epochs.

In addition to the aforementioned methods, numerous other MCDM methods have been suc-

successfully applied in studies conducted worldwide across a wide range of application domains. These include, for example, the PIV, PARIS, MARCOS, and COPRAS methods. Some recent studies demonstrating the application of these methods include: Yadav et al. [15], Trung [16], Kumar et al. [17], Mai [18], Ersoy [19] (applications of the PIV method); Ardil [20, 21], Le [22] (applications of the PARIS method); Basuri et al. [23] [24], Palanisamy et al. [25], Kaya and Yöndem [26], Trung [27] [28] (applications of the MARCOS method); and Adhikari et al. [29], Gazi et al. [30], Khan and Ali [31], Mishra et al. [32], Perdana et al. [33] (applications of the COPRAS method).

1.2 Research Motivation and Novelty

A review of the available literature indicates that, to the best of the author's knowledge, the PIV [34], PARIS [35], MARCOS [36] and COPRAS [37] methods have not previously been applied to the optimization of geodetic control networks. This research gap, together with the discussion presented in Subsection 1.1, motivated the present study to examine the applicability of these MCDM methods to this class of engineering geodesy problems and to compare their performance under different preference structures.

The novelty of this study lies in the introduction of the PIV, PARIS, MARCOS and COPRAS methods to the optimization of geodetic control networks, thereby extending the range of MCDM techniques available for this class of engineering geodesy problems. Furthermore, unlike previous studies ([1], [5], [7] and [12]), which focused on selecting the most appropriate geodetic network configuration or observation plan, the present research addresses a different optimization objective, namely the determination of the measurement accuracy required for reliable and efficient geodetic monitoring of engineering structure displacements. Consequently, the objective is not to optimize the geodetic network itself, but rather to determine the measurement accuracy level that ensures reliable displacement monitoring.

In addition to this novelty, the methodological contribution of this study lies in the development of a decision-making framework that integrates the SWING weighting method with four established MCDM methods (PIV, PARIS, MARCOS and COPRAS). The framework systematically evaluates four different preference approaches, compares the ranking results obtained using different MCDM methods, and examines the stability of the obtained rankings by means of a leave-one-out sensitivity analysis. This integrated evaluation procedure provides a comprehensive basis for assessing the consistency, stability and reliability of decision-making results under different preference structures.

1.3 Outline of the Research

Based on the considerations presented above, this subsection outlines the overall research procedure adopted in the study.

To establish the set of alternative solutions, six geodetic measuring instruments (total stations) with different measurement accuracies for horizontal directions and distances are considered. Each instrument defines one alternative, and all alternatives are evaluated within the same geodetic control network.

The data source for the decision-making process consist of the values of the criterion functions (see Table 2 in Subsection 3.1). These values are derived from the preliminary accuracy and reliability analysis of a bridge geodetic control network with a predefined (fixed) configuration, that is, from the least-squares adjustment performed during the network design stage. The adjustment procedure is carried out in accordance with the adjustment theory briefly presented in Subsection ??.

The criterion functions represent eight competitive and conflicting criteria related to the precision and reliability of geodetic measurements performed within the network (see Subsection 2.2). Their relative importance is expressed through four different preference approaches, each represented by an ordered 8-tuple of criterion weights derived using the SWING method (see Subsection 2.4). Within each preference approach, the most preferred criteria are assigned the highest weights, whereas the least preferred criteria receive the lowest weights.

As noted in Subsection 1.2, the study evaluates the applicability of the PIV, PARIS, MARCOS and COPRAS methods to the considered optimization problem. In addition, the TOPSIS method is employed as a reference benchmark to facilitate a comparative evaluation of the obtained ranking results.

Finally, the ranking results produced by the considered MCDM methods are analysed through comparative analysis, rank correlation assessment, and leave-one-out sensitivity analysis in order to evaluate the consistency, stability and reliability of the obtained decision outcomes.

2. Materials and Methodology Used in the Study

In this section, general characteristics of the system to be optimized in the study are presented first. Then, the criterion functions (criteria) which the alternative solutions are based on are given with the corresponding restrictions in detail. Also, the author provides the description for each of the six alternatives established.

Following that, the mathematical foundation of the PIV, PARIS, MARCOS, COPRAS, and TOPSIS methods is presented. Below, the method for determining criterion weights is introduced, as well as the metric for inferring the performance of the PIV, PARIS, MARCOS, and COPRAS methods used in the study.

2.1 General Characteristics of the System to be Optimized

The bridge geodetic control network used in the study (hereinafter: network), that is, actually, the system to be optimized herein, has these features: number of reference points, $n_R = 6$ (materialized by the centers of the pillars and denoted as R1, R2, ..., R6); number of control points discretizing the body of the bridge, $n_C = 42$ (materialized by bolt heads and denoted as: 1, 2, ..., 42); number of measured horizontal directions, $n_{hd} = 95$; number of measured distances, $n_d = 147$; total number of measurements, $n_m = n_{hd} + n_d = 242$; number of unknown orientations in the network, $u_o = 6$; number of unknown coordinates, $u_c = 2n_R + 2n_C = 96$; total number of unknowns, $u_t = u_o + u_c = 102$; network datum defect, $d = 3$.

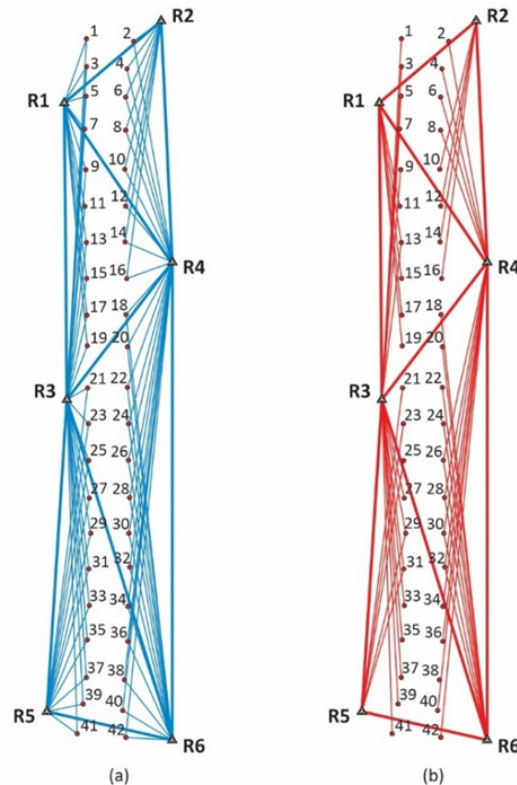


Fig. 1. Separately shown: (a) measured distances; and (b) measured horizontal directions in the network

It is important to mention that the configuration of the network, that is shown in Figure 1, in-

cludes some both-directions measurements between reference points, as well as some double measurements, both between reference points and towards control points.

As for the other general characteristics of the network, it is worth mentioning that: the network spread in the east-west and south-north direction is approximately 51.7 m and 296.5 m, respectively; minimal and maximal values of reference-to-reference point distance to be measured are approximately 52.3 m and 197.1 m, respectively; minimal and maximal values of reference-to-control point distance to be measured are approximately 9.2 m and 197.4 m, respectively.

All horizontal directions in the network are planned to be measured in two gyri. Simultaneously, the corresponding distances are read twice in each gyrus, which means the final distance measurement values are obtained based on four readings.

2.1.1.1 Network adjustment: A brief overview

The network adjustment is executed via the well-established method of least squares. Although a concise summary of this methodology is provided herein, a comprehensive exposition is available.

The computation begins with the standard construction of the design matrix ($\mathbf{A}_{n_m \times u_t}$) and the weight matrix ($\mathbf{P}_{n_m \times n_m}$). These matrices are calculated as follows:

$$\mathbf{A}_{n_m \times u_t} = \begin{bmatrix} \frac{\partial F_1}{\partial y_{R1}} & \frac{\partial F_1}{\partial x_{R1}} & \cdots & \frac{\partial F_1}{\partial y_{R6}} & \frac{\partial F_1}{\partial x_{R6}} & \frac{\partial F_1}{\partial y_1} & \frac{\partial F_1}{\partial x_1} & \cdots & \frac{\partial F_1}{\partial y_{42}} & \frac{\partial F_1}{\partial x_{42}} & \frac{\partial F_1}{\partial \varphi_{R1}} & \cdots & \frac{\partial F_1}{\partial \varphi_{R6}} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_{n_m}}{\partial y_{R1}} & \frac{\partial F_{n_m}}{\partial x_{R1}} & \cdots & \frac{\partial F_{n_m}}{\partial y_{R6}} & \frac{\partial F_{n_m}}{\partial x_{R6}} & \frac{\partial F_{n_m}}{\partial y_1} & \frac{\partial F_{n_m}}{\partial x_1} & \cdots & \frac{\partial F_{n_m}}{\partial y_{42}} & \frac{\partial F_{n_m}}{\partial x_{42}} & \frac{\partial F_{n_m}}{\partial \varphi_{R1}} & \cdots & \frac{\partial F_{n_m}}{\partial \varphi_{R6}} \end{bmatrix} \quad (1)$$

with F_i , $i \in \{1, 2, \dots, n_m\}$, representing the function related to the i^{th} measurement in the network (horizontal direction or distance), and $y_{R1}, x_{R1}, \dots, y_{R6}, x_{R6}, y_1, x_1, \dots, y_{42}, x_{42}, \varphi_{R1}, \dots, \varphi_{R6}$, that denote all unknown parameters in the network (coordinates of reference points, coordinates of control points and orientations at reference points), and

$$\mathbf{P}_{n_m \times n_m} = \text{diag} \left(\frac{\sigma_0^2}{\sigma_i^2} \right)_{i=1}^{n_m} \quad (2)$$

whereby σ_0^2 represents the adopted a priori variance, and σ_i^2 is the variance associated with the i^{th} measurement.

Matrices $\mathbf{A}_{n_m \times u_t}$ and $\mathbf{P}_{n_m \times n_m}$ are then utilized to form the singular normal equation coefficient matrix, defined as:

$$\mathbf{N}_{u_t \times u_t} = \mathbf{A}_{n_m \times u_t}^T \mathbf{P}_{n_m \times n_m} \mathbf{A}_{n_m \times u_t} \quad (3)$$

To proceed with the adjustment, the Moore-Penrose pseudoinverse $\mathbf{N}_{u_t \times u_t}^+$ must be evaluated, which serves as the cofactor matrix for the estimated parameters. Therefore, it follows that $\mathbf{Q}_{\hat{x}_{u_t \times u_t}} = \mathbf{N}_{u_t \times u_t}^+$. To compute, or more accurately, extract this pseudoinverse, one must invert the augmented block matrix formed by bordering the normal equation coefficient matrix with the datum conditions matrix of the free 2D network, i.e. the constraint matrix ($\mathbf{C}_{2D}$), yielding:

$$\begin{bmatrix} \mathbf{N}_{u_t \times u_t} & \mathbf{C}_{2D_{u_t \times 3}} \\ \mathbf{C}_{2D_{u_t \times 3}}^T & \mathbf{0}_{3 \times 3} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{N}_{u_t \times u_t}^+ & \mathbf{C}_{2D_{u_t \times 3}} (\mathbf{C}_{2D_{u_t \times 3}}^T \mathbf{C}_{2D_{u_t \times 3}})^{-1} \\ (\mathbf{C}_{2D_{u_t \times 3}}^T \mathbf{C}_{2D_{u_t \times 3}})^{-1} \mathbf{C}_{2D_{u_t \times 3}} & \mathbf{0}_{3 \times 3} \end{bmatrix} \quad (4)$$

This study utilizes two additional cofactor matrices. One of them is the cofactor matrix for the estimated observations (measured distances and horizontal directions), namely:

$$\mathbf{Q}_{i_{n_m \times n_m}} = \mathbf{A}_{n_m \times u_t} \mathbf{Q}_{\hat{x}_{u_t \times u_t}} \mathbf{A}_{n_m \times u_t}^T \quad (5)$$

and the another is the cofactor matrix for the residuals, that is:

$$\mathbf{Q}_{\hat{v}_{n_m \times n_m}} = \mathbf{P}_{n_m \times n_m}^{-1} - \mathbf{Q}_{i_{n_m \times n_m}} \quad (6)$$

Finally, the redundancy matrix is derived via

$$\mathbf{R}_{n_m \times n_m} = \mathbf{Q}_{\hat{v}_{n_m \times n_m}} \mathbf{P}_{n_m \times n_m} = \mathbf{I}_{n_m \times n_m} - \mathbf{A}_{n_m \times u_t} \mathbf{Q}_{\hat{x}_{u_t \times u_t}} \mathbf{A}_{n_m \times u_t}^T \quad (7)$$

where $\mathbf{I}_{n_m \times n_m}$ is identity matrix.

2.2 Defining the Criteria and the Alternative Solutions

Let's first introduce index $i \in \{1, 2, \dots, n_m\}$ to indicate the ordinal number of a measurement in the network and index $j \in \{1, 2, \dots, n_C\}$ for a control point. Now, the criteria for the purpose of this study can be listed. So, they are the following:

1. Sum of the cofactors of the control points' unknown Y and X coordinates (F1):

$$\sum_{j=1}^{n_C} tr \mathbf{Q}_{\hat{x}_{j2 \times 2}} = \sum_{j=1}^{n_C} (Q_{\hat{y}_j \hat{y}_j} + Q_{\hat{x}_j \hat{x}_j}) \quad (8)$$

2. Maximal ratio of semi-major to semi-minor axis of the control point's standard error ellipse (F2):

$$\max_j \left(\frac{A}{B} \right)_j = \max_j \left(\sqrt{\frac{Q_{\hat{y}_j \hat{y}_j} + Q_{\hat{x}_j \hat{x}_j} + \sqrt{(Q_{\hat{y}_j \hat{y}_j} - Q_{\hat{x}_j \hat{x}_j})^2 + 4Q_{\hat{y}_j \hat{x}_j}^2}}{Q_{\hat{y}_j \hat{y}_j} + Q_{\hat{x}_j \hat{x}_j} - \sqrt{(Q_{\hat{y}_j \hat{y}_j} - Q_{\hat{x}_j \hat{x}_j})^2 + 4Q_{\hat{y}_j \hat{x}_j}^2}}} \right) \quad (9)$$

3. Maximal standard error of control point's position (F3):

$$\max_j (m_{p_j}) = \max_j \left(\sigma_0 \sqrt{Q_{\hat{y}_j \hat{y}_j} + Q_{\hat{x}_j \hat{x}_j}} \right) \quad (10)$$

4. Minimal redundancy coefficient for the entire network (F4):

$$\min_i (r_{ii}) = \min_i (\mathbf{R}_{n_m \times n_m})_{ii} \quad (11)$$

5. Sum of detectable marginal gross-error values (not considering the units of measurement) in measured horizontal directions and distances (indices i_{hd} and i_d are introduced for the measured horizontal directions and distances, respectively) (F5):

$$\sum_{i_{hd}=1}^{n_{hd}} \frac{|G_{i_{hd}}^*|}{''} + \sum_{i_d=1}^{n_d} \frac{|G_{i_d}^*|}{\text{mm}} = \sum_{i_{hd}=1}^{n_{hd}} \frac{(t_{1-\beta_0} + t_{1-\alpha_0/2}) \sigma_{i_{hd}} / ''}{\sqrt{r_{i_{hd} i_{hd}}}} + \sum_{i_d=1}^{n_d} \frac{(t_{1-\beta_0} + t_{1-\alpha_0/2}) \sigma_{i_d} / \text{mm}}{\sqrt{r_{i_d i_d}}} \quad (12)$$

6. Sum of influences on adjusted observations (F6):

$$\frac{1}{n_m} tr (\mathbf{P}_{n_m \times n_m} \mathbf{Q}_{i_{n_m \times n_m}} \mathbf{P}_{n_m \times n_m}) = \frac{1}{n_m} \sum_{i=1}^{n_m} (P_i^2 Q_{\hat{l}_i \hat{l}_i}) \quad (13)$$

7. Average value of the Cook-Perović's distance (F7):

$$\overline{CP} = \frac{1}{n_m} \sum_{i=1}^{n_m} CP_i = \frac{1}{n_m} \sum_{i=1}^{n_m} \left(\frac{t_{1-\alpha_0/2}^2}{r(\mathbf{Q}_{\hat{\mathbf{x}}_{u_t \times u_t}})} \cdot \frac{1 - r_{ii}}{r_{ii}} \right) \quad (14)$$

8. Average value of the minimal control point displacement between two epochs that can be reliably detected (F8):

$$\overline{d_p} = \frac{1}{n_C} \sum_{j=1}^{n_C} d_{p_j} = \frac{1}{n_C} \sum_{j=1}^{n_C} \left(\frac{\sigma_0 \sqrt{\lambda}}{\sqrt{\mathbf{p}_{j_{u_t \times 1}}^T \mathbf{Q}_{d_{u_t \times u_t}}^+ \mathbf{p}_{j_{u_t \times 1}}}} \right) \quad (15)$$

whereby, except those introduced, the following notations are present: $\sigma_{i_{hd}}$ and σ_{i_d} – measurement accuracy for horizontal directions and distances, respectively; $1 - \beta_0$ and α_0 – test power and significance level in hypotheses testing, respectively; λ – noncentrality parameter read from the corresponding table, based on function of test power, significance level and the (2D network) figure rank $r_f = u_c - 3$; $\mathbf{Q}_{\hat{\mathbf{x}}_{j_2 \times 2}}$ – submatrix of the cofactor matrix matrix $\mathbf{Q}_{\hat{\mathbf{x}}_{u_t \times u_t}}$ related to the control point j ; $Q_{(\blacksquare)}$ – cofactors for $(\blacksquare)$; $\mathbf{p}_{j_{u_t \times 1}}^T = \begin{bmatrix} \mathbf{0}_{2(j-1) \times 1}^T & \sin \theta_j & \cos \theta_j & \mathbf{0}_{2(n_C-j) \times 1}^T \end{bmatrix}$ – the vector corresponding to the control point j ; θ_j – the standard error ellipse azimuth angle obtained from $\tan 2\theta_j = 2Q_{\hat{y}_j \hat{x}_j} / (Q_{\hat{x}_j \hat{x}_j} - Q_{\hat{y}_j \hat{y}_j})$; and $\mathbf{Q}_{d_{u_t \times u_t}}^+$ – pseudoinverse of the matrix $\mathbf{Q}_{d_{u_t \times u_t}} = 2\mathbf{Q}_{\hat{\mathbf{x}}_{u_t \times u_t}}$.

The criterion F4 represents a gain, so it is to be maximized. On the other hand, all the remaining criteria (F1, F2, F3, F5, F6, F7 and F8) are to be minimized, since they are loss-oriented.

Prior to establishing alternatives as acceptable solutions, the following constraints that must be fulfilled are introduced:

$$\frac{A_r}{B_r} \leq 2.2 \quad (16)$$

$$\frac{A_j}{B_j} \leq 2.5 \quad (17)$$

$$m_{p_j} \leq 3.5 \quad (18)$$

$$r_{ii} \geq 0.20 \quad (19)$$

$$|G_{i_{hd}}^*| \leq \frac{t_{0.80} + t_{1-0.001/2}}{\sqrt{0.20}} \sigma_{i_{hd}} \approx 9.24 \sigma_{i_{hd}} \quad (20)$$

$$|G_{i_d}^*| \leq \frac{t_{0.80} + t_{1-0.001/2}}{\sqrt{0.20}} \sigma_{i_d} \approx 9.24 \sigma_{i_d} \quad (21)$$

$$d_{p_j} \leq 25 \text{ mm} \quad (22)$$

where $i \in \{1, 2, \dots, n_m\}$, $j \in \{1, 2, \dots, n_C\}$, $r \in \{R1, R2, R3, R4, R5, R6\}$, $i_{hd} \in \{1, 2, \dots, n_{hd}\}$ and $i_d \in \{1, 2, \dots, n_d\}$.

Six acceptable solutions (alternatives), each of which involves the use of a geodetic instrument (total station) with specific measurement precision, are established to fulfill all the constraints given by Eqs. (16-22). Hence, those alternatives are the subject of the analysis in this study. All of them are listed in Table 1.

Table 1

Geodetic instruments (total stations) used in establishing the alternatives to be ranked herein

Alternative	Total station	Measurement accuracy		Measurement mode	
		Horizontal direction (one gyrus)	Distance	Between reference points	Towards control points
A1	Leica FlexLine TS09	2"	1 mm + 1.5 ppm 2 mm + 2 ppm	Prism	Reflectorless
A2	Leica FlexLine TS09	3"	1 mm + 1.5 ppm 2 mm + 2 ppm	Prism	Reflectorless
A3	Leica FlexLine TS06	2"	1.5 mm + 2 ppm 2 mm + 2 ppm	Prism	Reflectorless
A4	Leica FlexLine TS06	3"	1.5 mm + 2 ppm 2 mm + 2 ppm	Prism	Reflectorless
A5	Trimble S7	3"	1 mm + 2 ppm 2 mm + 2 ppm	Prism	Reflectorless
A6	Topcon OS-103	3"	2 mm + 2 ppm 3 mm + 2 ppm	Prism	Reflectorless

2.3 Mathematical Background of the MCDM Methods Used in the Study

In this section, the mathematical basis of the PIV, PARIS, MARCOS, COPRAS and TOPSIS methods is given. The presentation provides a shortened relevant content.

2.3.1 The PIV method

This method was proposed by Mufazzal & Muzakkir [34]. The basic idea was to prevent the rank reversal phenomenon. The PIV method is carried out through the following steps:

Step 1. Evaluation of each of m alternatives by each of n criteria, i.e., establishing the decision matrix (denoted as **F** herein) using original data as follows:

$$\mathbf{F}_{m \times n} = (f_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} = \begin{bmatrix} f_{11} & \dots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \dots & f_{mn} \end{bmatrix} \quad (23)$$

Step 2. Determination of the normalized decision matrix (denoted as **R** herein) according to:

$$\mathbf{R}_{m \times n} = (r_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} = \begin{bmatrix} r_{11} & \dots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \dots & r_{mn} \end{bmatrix} \quad (24)$$

where $r_{ij} = f_{ij} / \sqrt{\sum_{i=1}^m f_{ij}^2}$, representing the normalized value associated to the alternative i and criterion j .

Step 3. Determination of the weighted normalized decision matrix (denoted as $\mathbf{V}$ herein) in the following way:

$$\mathbf{V}_{m \times n} = (v_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} = \begin{bmatrix} v_{11} & \dots & v_{1n} \\ \vdots & \ddots & \vdots \\ v_{m1} & \dots & v_{mn} \end{bmatrix} \quad (25)$$

where $v_{ij} = w_j r_{ij}$, with w_j representing the weight of the criterion j .

Step 4. Determination of the weighted proximity indices' matrix (denoted as $\mathbf{E}$ herein) as shown below:

$$\mathbf{E}_{m \times n} = (e_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} = \begin{bmatrix} e_{11} & \dots & e_{1n} \\ \vdots & \ddots & \vdots \\ e_{m1} & \dots & e_{mn} \end{bmatrix} \quad (26)$$

whereby for a gain-oriented criterion ($i \in \{1, \dots, m\}, j \in J' = \{1, \dots, n'\}$):

$$e'_{ij} = \max_i(v_{ij}) - v_{ij} \quad (27)$$

and for a loss-oriented criterion ($i \in \{1, \dots, m\}, j \in J'' = \{n' + 1, \dots, n\}$):

$$e''_{ij} = v_{ij} - \min_i(v_{ij}) \quad (28)$$

where J' and J'' represent, respectively, the sets associated to gain-oriented criteria (assuming these criteria are the first n' of all criteria in the list) and loss-oriented criteria (the remaining $(n - n')$ ones in the list).

Step 5. Calculation of the overall proximity values for each alternative as follows:

$$d_i = \sum_{j=1}^{n'} e'_{ij} + \sum_{j=n'+1}^n e''_{ij} \quad (29)$$

with $i \in \{1, \dots, m\}$ and j as in Eqs. (27) and (28).

Step 6. Ranking the alternatives in such a way that the one with the lowest value of d_i is declared the best.

2.3.2 The PARIS method

The method was proposed by Ardil [35] and the basic idea was to compare relative distances from competing alternatives to a reference ideal point. The PARIS method involves the following steps:

Step 1. The same as Step 1 of the PIV method.

Step 2. Determination of the normalized decision matrix $\mathbf{R} = [r_{ij}]_{m \times n}$, consisting of elements calculated as follows:

$$r'_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^m f_{ij}^2}} \quad (30)$$

for a gain-oriented criterion ($i \in \{1, \dots, m\}, j \in J' = \{1, \dots, n'\}$), and

$$r''_{ij} = 1 - \frac{f_{ij}}{\sqrt{\sum_{i=1}^m f_{ij}^2}} \quad (31)$$

for a loss-oriented criterion ($i \in \{1, \dots, m\}, j \in J'' = \{n' + 1, \dots, n\}$), whereby J' and J'' were introduced with Eqs. (27) and (28).

Step 3. Determination of the weighted normalized matrix (denoted as $\mathbf{Z}$) using the values r'_{ij} and r''_{ij} , calculated according to Eqs. (30) and (31). So, the elements of the matrix $\mathbf{Z}$ are the following:

$$z'_{ij} = w_j r'_{ij} \quad (32)$$

for a gain-oriented criterion ($i \in \{1, \dots, m\}, j \in J' = \{1, \dots, n'\}$), and

$$z''_{ij} = w_j r''_{ij} \quad (33)$$

for a loss-oriented criterion ($i \in \{1, \dots, m\}, j \in J'' = \{n' + 1, \dots, n\}$).

Step 4. Calculation of the following weighted sum using the following formula:

$$\pi_i^w = \sum_{j=1}^{n'} z'_{ij} + \sum_{j=n'+1}^n z''_{ij} \quad (34)$$

with $i \in \{1, \dots, m\}$ and j as in Eqs. (30) and (31).

Step 5. Ranking the alternatives according to decreasing values of π_i^w . So, the one with the highest appraisal score is the best choice among all competing alternatives.

Step 6. Determination of the elements of the reference ideal solution (denoted as Z^*) as follows:

$$Z^* = \{z_1^*, \dots, z_n^*\} = \{(\max_i(z'_{ij}) \mid j \in J'), (\max_i(z''_{ij}) \mid j \in J'')\} \quad (35)$$

(Remark: By introducing the normalized values as given by Eqs. (30) and (31), the problem boils down to the maximization of both z'_{ij} and z''_{ij} quantities.)

Step 7. Calculation of the distance from the reference ideal solution for each alternative (denoted as π_i^*), according to:

$$\pi_i^* = \sum_{j=1}^{n'} (z_j^* - z'_{ij}) + \sum_{j=n'+1}^n (z_j^* - z''_{ij}) \quad (36)$$

where $i \in \{1, \dots, m\}$, and j is the same as in Eq. (34).

Step 8. Ranking the alternatives according to increasing values of π_i^* . The alternative associated to the lowest appraisal score is the best one.

Step 9. Calculation of the relative distance from each alternative to the reference ideal point (denoted as R_i), using the following formula:

$$R_i = \sqrt{(\pi_i^w - \max_i(\pi_i^w))^2 + (\pi_i^* - \min_i(\pi_i^*))^2} \quad (37)$$

where $i \in \{1, \dots, m\}$.

Step 10. Ranking the alternatives in such a way that the one with the lowest value of R_i is declared the best.

2.3.3 The MARCOS method

This method was proposed by Stević et al. [36] and is based on defining the relationship between alternatives and reference values (ideal and anti-ideal alternatives). In decision-making, utility functions play a key role. The MARCOS method is implemented through the following steps:

Step 1. The same as Step 1 of the PIV method.

Step 2. Calculation of the extended decision matrix (denoted as $\mathbf{F}'$ herein) by adding elements of an ideal alternative (AI) and anti-ideal alternative (AAI):

$$\mathbf{F}'_{(m+2) \times n} = \begin{bmatrix} f_{AAI1} & \cdots & f_{AAIn} \\ f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \\ f_{AI1} & \cdots & f_{AI n} \end{bmatrix} \quad (38)$$

where for a gain-oriented criterion ($i \in \{1, \dots, m\}, j \in J' = \{1, \dots, n'\}$)

$$f_{AIj} = \max_i(f_{ij}) \text{ and } f_{AAIj} = \min_i(f_{ij}) \quad (39)$$

where for a loss-oriented criterion ($i \in \{1, \dots, m\}, j \in J'' = \{n' + 1, \dots, n\}$)

$$f_{AIj} = \min_i(f_{ij}) \text{ and } f_{AAIj} = \max_i(f_{ij}) \quad (40)$$

whereby J' and J'' were introduced with Eqs. (27) and (28).

Step 3. Determination of the normalized decision matrix $\mathbf{N} = [n_{ij}]_{m \times n}$ with elements calculated as follows:

$$n_{ij} = \frac{f_{ij}}{f_{AIj}} \quad (41)$$

with $i \in \{1, \dots, m\}, j \in J' = \{1, \dots, n'\}$, for a gain-oriented criterion, and

$$n_{ij} = \frac{f_{AIj}}{f_{ij}} \quad (42)$$

with $i \in \{1, \dots, m\}, j \in J'' = \{n' + 1, \dots, n\}$, for a loss-oriented criterion.

Step 4. Forming the weighted normalized decision matrix $\mathbf{V} = [v_{ij}]_{m \times n}$ based on its elements $v_{ij} = w_j n_{ij}$, where w_j represents the weight of the criterion j .

Step 5. Evaluation of the utility degrees of an alternative in relation to the ideal and anti-ideal solution (denote as K_i^+ and K_i^- , respectively) using the following formulas:

$$K_i^+ = \frac{S_i}{S_{AI}} \quad (43)$$

$$K_i^- = \frac{S_{AI}}{S_i} \quad (44)$$

where $S_i = \sum_{j=1}^n v_{ij}$, $S_{AI} = \sum_{j=1}^n f_{AIj}$ and $S_{AAI} = \sum_{j=1}^n f_{AAIj}$.

Step 6. Calculating the values of the utility functions in relation to the ideal and anti-ideal solution (denoted as $f(K_i^+)$ and $f(K_i^-)$, respectively) in the following way:

$$f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^-} \quad (45)$$

$$f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^-} \quad (46)$$

Step 7. Calculating the value of the utility function (denoted as $f(K_i)$ herein), for each alternative as follows:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1-f(K_i^+)}{f(K_i^+)} + \frac{1-f(K_i^-)}{f(K_i^-)}} \quad (47)$$

Step 8. Ranking the alternatives based on the rule that the one with the highest value of $f(K_i)$ is considered the best.

2.3.4 The COPRAS method

This method was introduced by Zavadskas and Kaklauskas in 1996, it was later presented in greater detail in Zavadskas et al. [37] and was utilized to assess the superiority of one option over another and enables the comparison of options. It systematically rates and assesses options based on their importance and utility level, and involves the following steps:

Step 1. The same as Step 1 of the PIV method.

Step 2. Determination of the normalized decision matrix $\mathbf{R} = [r_{ij}]_{m \times n}$, consisting of elements calculated as $r_{ij} = f_{ij} / \sqrt{\sum_{i=1}^m f_{ij}^2}$, with $i \in \{1, \dots, m\}$, $j \in \{1, \dots, n\}$.

Step 3. Determination of the weighted normalized decision matrix $\mathbf{V} = [v_{ij}]_{m \times n}$, whereby its elements are $v_{ij} = w_j r_{ij}$, with w_j representing the weight of the criterion j .

Step 4. Calculating the maximizing and minimizing indices (denoted as S_{+i} and S_{-i} , respectively) using the following formulas:

$$S_{+i} = \sum_{j=1}^{n'} v_{ij} \quad (48)$$

for a gain-oriented criterion ($i \in \{1, \dots, m\}$, $j \in J' = \{1, \dots, n'\}$), and

$$S_{-i} = \sum_{j=n'+1}^n v_{ij} \quad (49)$$

for a loss-oriented criterion ($i \in \{1, \dots, m\}$, $j \in J'' = \{n' + 1, \dots, n\}$), with J' and J'' introduced with Eqs. (27) and (28).

Step 5. Calculation of the relative weight as follows:

$$Q_i = S_{+i} + \frac{\min_i S_{-i} \sum_{i=1}^m S_{-i}}{S_{-i} \sum_{i=1}^m \frac{\min_i S_{-i}}{S_{-i}}} \quad (50)$$

with $i \in \{1, \dots, m\}$.

Step 6. Ranking the alternatives in such a way that the one having the highest Q_i value is the best choice.

2.3.5 The TOPSIS method

It is about the method that was initially proposed by Hwang and Yoon [4]. The basic idea for developing this method was to find a mathematical tool that will provide the optimal alternative being as close in distance as possible to a positive ideal solution and, at the same time, as far away as possible from the corresponding negative ideal solution. The TOPSIS method involves the following steps:

Step 1. The same as Step 1 of the PIV method.

Step 2. The same as Step 2 of the PIV method.

Step 3. The same as Step 3 of the PIV method.

Step 4. Derivation of the positive ideal solution (denoted as A^+) and negative ideal solution (denoted as A^-) in the following way:

$$A^+ = \{v_1^+, \dots, v_n^+\} = \{(\max_i(v_{ij}) \mid j \in J'), (\min_i(v_{ij}) \mid j \in J'')\} \quad (51)$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \{(\min_i(v_{ij}) \mid j \in J'), (\max_i(v_{ij}) \mid j \in J'')\} \quad (52)$$

where $i \in \{1, \dots, m\}$, j is as in Eqs. (27) and (28), and v_{ij} was previously introduced by Eq. (25).

Step 5. Calculation of two n —dimensional Euclidean distances for each alternative i , one of which is the distance from the positive ideal solution (denoted as D_i^+), and the another one is the distance from the negative ideal solution (denoted as D_i^-), using the following formulas:

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (53)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (54)$$

with $i \in \{1, \dots, m\}$, $j \in \{1, \dots, n\}$.

Step 6. Calculation of the relative closeness of each alternative to the ideal solution according to:

$$C_i = D_i^- / (D_i^+ + D_i^-), 0 \leq C_i \leq 1 \quad (55)$$

with $i \in \{1, \dots, m\}$.

Step 7. Ranking the alternatives according to the principle that the one with the highest value of C_i is declared the best.

2.4 The Method for Determining Criterion Weights Used in the Study

There is a number of methods used for determining criterion weights in MCDM tasks that many papers deal with. Some of them are the following: Simple Multi-Attribute Rating Technique (SMART) [38] [39], Analytic Hierarchy Process (AHP) [2], Rank Summed (RS) Weighting Technique [40], Rank Reciprocal (RR) Weighting Technique [40], Swing (SWING) Weighting Technique [39], Rank Ordered Centroid (ROC) Technique [41], Step-Wise Weight Assessment Ratio Analysis (SWARA) [42], etc.

In this study, the SWING method is used. This method is based on the following sequence of procedures:

1. Subjective ranking of criteria with respect to importance, i.e. according to a preference.
2. Assigning the greatest number of points to the most important criterion.
3. Assignment of numbers of points to each of the remaining criteria in such a way to represent the degree of importance of a criterion relative to the reference, i.e. to the most important criterion.
4. Calculation of criterion weight value for each criterion as the ratio of the number of points assigned to that criterion to the total number of points.

The author uses four preference approaches, denoted as Preference Approach I, II, III and IV hereinafter. The distributions of the numbers of points assigned to the eight criteria (F1, F2, ..., F8) by each of the four approaches are the following:

- Preference Approach I: 1, 1, 1, 1, 1, 1, 1, 1;
- Preference Approach II: 4, 7, 5, 4, 6, 5, 6, 12;
- Preference Approach III: 3, 3, 6, 6, 4, 3, 3, 6;
- Preference Approach IV: 5, 4, 4, 4, 6, 2, 2, 6.

As can be seen, in the first preference approach no criterion has priority over another, while that is not the case with the remaining ones. Namely, in the second one a priority is given to criterion F8, which means that it is desirable the network provides the minimal average value of the minimal control point displacement between two epochs that can be reliably detected. When it comes to the third and fourth approaches, the preferences are highlighted through the criterion weights given to criteria F3, F4 and F8 (for Preference Approach III), i.e. F1, F5 and F8 (for Preference Approach IV).

2.5 Metric for Inferring the Performance of the PIV, PARIS, MARCOS and COPRAS Methods

In order to assess the stability of alternative rankings obtained using different MCDM methods, the author uses the Spearman rank correlation coefficient as an appropriate metric.

This coefficient (denoted as $S_{\mu, TOPSIS}$) is calculated as follows [43]:

$$S_{\mu, TOPSIS} = 1 - \frac{6 \sum_{i=1}^m D_i^2}{m(m^2 - 1)}, \quad -1 \leq S_i \leq +1 \quad (56)$$

with $i \in \{1, \dots, m\}$, where

$$D_i = \text{rank}(A_{i, \mu}) - \text{rank}(A_{i, TOPSIS}) \quad (57)$$

representing the difference in rank for alternative A_i when ranked by particular MCDM method μ (PIV, PARIS, MARCOS, COPRAS), and by the TOPSIS method, that was chosen as a reference benchmark in this study due to its very wide and successful application in practice. In addition, by many authors, the TOPSIS method is declared one of the top five, even three, widely-used MCDM methods in engineering practice.

The alternative ranking correlation between two MCDM methods is considered significant if $|S_i| \geq S_{crit}(m; \alpha)$, where $S_{crit}(m; \alpha)$ is the corresponding critical value that depends on the number of alternatives (m) and the significance level (α) (in this study, $\alpha = 0.05$ is adopted), and is taken from the rank correlation coefficient critical values table (valid for $m < 30$, which fully corresponds to the number of alternatives in this study). For this study, $S_{crit}(6; 0.05) = 0.886$.

3. Results and Discussion

This section presents and discusses the results obtained by applying the proposed decision-making framework. First, the values of the eight criteria are determined for each alternative, and the corresponding initial ranking lists are established. Subsequently, the normalized criterion values and criterion weights used in the final calculations as well as the final results of applying the five methods are presented. The alternative rankings obtained using the considered MCDM methods are then analysed through comparative analysis and rank correlation assessment. Finally, the stability and reliability of the obtained rankings are examined by means of a leave-one-out sensitivity analysis.

3.1 Evaluation of Alternatives, Initial Ranking Lists, Normalized Criterion Values and Weights Used in the Final Calculations

The results of evaluation of the six alternatives ($A_1, A_2, \dots, A_6$) by each of the eight criteria (F1, F2, $\dots$, F8) are shown in Table 2.

As previously said in Subsection 2.2, only the criterion F4 represents a gain (thus, it is to be maximized) and all the remaining criteria represent losses (they are to be minimized).

An analysis of the data provided in Table 2 indicates that all alternatives constitute fairly close acceptable solutions. This outcome was anticipated due to the atypical nature of the network, which

Table 2
Alternative evaluation using the criteria

Criterion	Alternative					
	A1	A2	A3	A4	A5	A6
F1	20.5064 mm ²	12.5201 mm ²	20.9342 mm ²	12.8601 mm ²	20.5580 mm ²	20.0733 mm ²
F2	2.179	2.124	2.130	2.136	2.128	2.109
F3	1.30 mm	1.67 mm	1.31 mm	1.69 mm	1.68 mm	1.92 mm
F4	0.295	0.315	0.289	0.312	0.315	0.293
F5	1299.37	1599.54	1319.16	1621.07	1601.69	1943.76
F6	0.6393	1.2848	0.5795	1.1259	1.2610	0.6119
F7	0.0504	0.0494	0.0509	0.0498	0.0494	0.0507
F8	6.802 mm	7.821 mm	6.800 mm	7.818 mm	7.820 mm	9.986 mm

necessitates high-precision measurements, i.e. total stations, and the use of ones with nearly identical technical specifications (see Table 1 in Subsection 2.2). It is worth noting that optimization becomes considerably more difficult when dealing with close alternatives rather than distinctly different ones.

Considering the values given in Table 2, initial ranking lists of the six alternatives are established for each of the eight criteria as presented in Table 3.

Table 3
Initial alternative rankings based on the criterion values

Criterion	Positions of the alternatives in the initial ranking lists					
	First	Second	Third	Fourth	Fifth	Sixth
F1	A2	A4	A6	A1	A5	A3
F2	A6	A2	A3	A4	A5	A1
F3	A1	A5	A3	A2	A4	A6
F4	A2	A4	A1	A5	A6	A3
F5	A1	A5	A3	A2	A4	A6
F6	A3	A6	A5	A1	A4	A2
F7	A2	A4	A1	A5	A6	A3
F8	A3	A5	A1	A4	A2	A6

As stated in Subsection 2.3, the PIV, PARIS, MARCOS and COPRAS methods use different normalized values of criteria, and the TOPSIS method uses the same ones as PIV. Therefore, four instead of five tables of normalized data are shown below (Tables 4, 5, 6 and 7).

Table 4
Obtained normalized criterion values used in the PIV and TOPSIS methods

Criterion	Normalized criterion values for each alternative					
	A1	A2	A3	A4	A5	A6
F1	0.49139	0.30001	0.50164	0.30816	0.30092	0.48101
F2	0.41684	0.40628	0.40730	0.40856	0.40703	0.40335
F3	0.32887	0.42483	0.33200	0.42892	0.42530	0.48609
F4	0.39642	0.42445	0.38942	0.42002	0.42389	0.39363
F5	0.33596	0.41357	0.34108	0.41914	0.41413	0.50257
F6	0.26949	0.54158	0.24428	0.47463	0.53158	0.25796
F7	0.41067	0.40258	0.41463	0.40565	0.40292	0.41288
F8	0.35093	0.40351	0.35085	0.40335	0.40348	0.51522

The 8-tuples of criterion weights for each of the five preference approaches mentioned previously

Table 5
Obtained normalized criterion values used in the PARIS method

Criterion	Normalized criterion values for each alternative					
	A1	A2	A3	A4	A5	A6
F1	0.50861	0.69999	0.49836	0.69184	0.69908	0.51899
F2	0.58316	0.59372	0.59270	0.59144	0.59297	0.59665
F3	0.67113	0.57517	0.66800	0.57108	0.57470	0.51391
F4	0.39642	0.42445	0.38942	0.42002	0.42389	0.39363
F5	0.66404	0.58643	0.65892	0.58086	0.58587	0.49743
F6	0.73051	0.45842	0.75572	0.52537	0.46842	0.74204
F7	0.58933	0.59742	0.58537	0.59435	0.59708	0.58712
F8	0.64907	0.59649	0.64915	0.59665	0.59652	0.48478

Table 6
Obtained normalized criterion values used in the MARCOS method

Criterion	Normalized criterion values for each alternative					
	A1	A2	A3	A4	A5	A6
F1	0.61055	1.00000	0.59807	0.97356	0.99698	0.62372
F2	0.96763	0.99277	0.99029	0.98723	0.99094	1.00000
F3	1.00000	0.77412	0.99059	0.76674	0.77327	0.67657
F4	1.01797	1.08995	1.00000	1.07858	1.08851	1.01081
F5	1.00000	0.81234	0.98500	0.80155	0.81125	0.66848
F6	0.90647	0.45106	1.00000	0.51468	0.45954	0.94699
F7	0.98032	1.00000	0.97095	0.99245	0.99917	0.97506
F8	0.99979	0.86951	1.00000	0.86986	0.86956	0.68098

Table 7
Obtained normalized criterion values used in the COPRAS method

Criterion	Normalized criterion values for each alternative					
	A1	A2	A3	A4	A5	A6
F1	0.20619	0.12589	0.21050	0.12931	0.12627	0.20184
F2	0.17018	0.16587	0.16629	0.16680	0.16618	0.16467
F3	0.13556	0.17512	0.13685	0.17680	0.17531	0.20036
F4	0.16195	0.17340	0.15909	0.17159	0.17317	0.16081
F5	0.13846	0.17044	0.14057	0.17274	0.17067	0.20712
F6	0.11618	0.23349	0.10532	0.20462	0.22918	0.11121
F7	0.16767	0.16437	0.16928	0.16562	0.16450	0.16857
F8	0.14457	0.16623	0.14454	0.16617	0.16622	0.21226

(see Subsection 2.4) are presented in Table 8.

3.2 Final Results of Applying the Five Methods

In this subsection, the final results of applying the PIV, PARIS, MARCOS, COPRAS and TOPSIS methods are presented.

The overall proximity values, d_i , $i \in \{1, \dots, m\}$, obtained by applying the PIV method, and the corresponding rankings of the alternatives are shown in Table 9.

It is readily apparent that, in each of the four preference approaches, the PIV method positioned alternatives A2, A4, A5, and A6 as fifth, third, fourth and sixth on the ranking list, respectively. Mean-

Table 8
Criterion weights under each preference approach

Criterion	Criterion weights			
	Preference Approach I	Preference Approach II	Preference Approach III	Preference Approach IV
F1	0.12500	0.08163	0.08824	0.15152
F2	0.12500	0.14286	0.08824	0.12121
F3	0.12500	0.10204	0.17647	0.12121
F4	0.12500	0.08163	0.17647	0.12121
F5	0.12500	0.12245	0.11765	0.18182
F6	0.12500	0.10204	0.08824	0.06061
F7	0.12500	0.12245	0.08824	0.06061
F8	0.12500	0.24490	0.17647	0.18182

Table 9
Final results of the PIV method for all preference approaches

Preference Approach	Final results	Final results					
		A1	A2	A3	A4	A5	A6
I	d_i	0.03328	0.06581	0.03261	0.06087	0.06500	0.09050
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
II	d_i	0.02342	0.06295	0.02230	0.05890	0.06230	0.09664
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
III	d_i	0.02597	0.06185	0.02654	0.05926	0.06138	0.09987
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th
IV	d_i	0.03606	0.05369	0.03731	0.05334	0.05355	0.11185
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th

while, alternatives A1 and A3 consistently emerged as the top two.

When it comes to PARIS method, the ranking of alternatives was established based on the weighted sums (π_i^w), the distance from the reference ideal solution for each alternative (π_i^*) and the relative distance from each alternative to the reference ideal point (R_i), with $i \in \{1, \dots, m\}$. The corresponding values and the rankings are given in Table 10.

An examination of Table 10 leads to the same conclusion as in the case of the PIV method.

The final results established by applying the MARCOS method, including the utility degrees of alternatives in relation to the ideal and anti-ideal solution (K_i^+ and K_i^- , respectively), the values of the utility function ($f(K_i)$), with $i \in \{1, \dots, m\}$, as well as the corresponding ranking lists, are shown in Table 11.

The ranking lists obtained using the MARCOS method lead to the conclusion that there are deviations from the rankings generated by the PIV and PARIS methods. Namely, in as many as three out of four preference approaches, the MARCOS method identified alternative A3 as the optimal choice, while alternatives A2 and A4 interchanged their positions on the ranking lists.

And, at the very end, the final results obtained by applying the COPRAS method are given (Table 12). It is about the the maximizing and minimizing indices (S_{+i} and S_{-i} , respectively), the relative weights (Q_i), with $i \in \{1, \dots, m\}$. The corresponding ranking lists follow these data.

Scrutiny of Table 12 corroborates the exact conclusion reached via the PIV and PARIS methods.

In applying the TOPSIS method, the ranking of the alternatives was established based on the obtained data including the distances from the positive ideal solution (D_i^+), the distances from the neg-

Table 10
Final results of the PARIS method for all preference approaches

Preference Approach	Final results	A1	A2	A3	A4	A5	A6
I	π_i^w	0.59904	0.56651	0.59971	0.57145	0.56732	0.54182
	π_i^*	0.03328	0.06581	0.03261	0.06087	0.06500	0.09050
	R_i	0.00095	0.04695	0.00000	0.03996	0.04581	0.08186
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
II	π_i^w	0.61264	0.57312	0.61376	0.57716	0.57376	0.53942
	π_i^*	0.02342	0.06295	0.02230	0.05890	0.06230	0.09664
	R_i	0.00157	0.05748	0.00000	0.05176	0.05657	0.10513
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
III	π_i^w	0.59385	0.55797	0.59328	0.56056	0.55843	0.51994
	π_i^*	0.02597	0.06185	0.02654	0.05926	0.06138	0.09987
	R_i	0.00000	0.05074	0.00080	0.04708	0.05008	0.10451
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th
IV	π_i^w	0.59589	0.57826	0.59463	0.57860	0.57839	0.52010
	π_i^*	0.03606	0.05369	0.03731	0.05334	0.05355	0.11185
	R_i	0.00000	0.02493	0.00177	0.02444	0.02474	0.10718
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th

Table 11
Final results of the MARCOS method for all preference approaches

Preference Approach	Final results	A1	A2	A3	A4	A5	A6
I	K_i^+	0.0007070	0.0006604	0.0007119	0.0006599	0.0006603	0.0006219
	K_i^-	0.0004723	0.0004412	0.0004756	0.0004409	0.0004412	0.0004155
	$f(K_i)$	0.0003726	0.0003481	0.0003752	0.0003478	0.0003480	0.0003278
	Ranked:	2 nd	3 rd	1st	5 th	4 th	6 th
II	K_i^+	0.0007203	0.0006593	0.0007252	0.0006591	0.0006593	0.0006122
	K_i^-	0.0004812	0.0004405	0.0004845	0.0004403	0.0004404	0.0004090
	$f(K_i)$	0.0003797	0.0003475	0.0003822	0.0003474	0.0003475	0.0003227
	Ranked:	2 nd	3 rd	1st	5 th	4 th	6 th
III	K_i^+	0.0007225	0.0006665	0.0007239	0.0006647	0.0006663	0.0006118
	K_i^-	0.0004827	0.0004453	0.0004836	0.0004441	0.0004451	0.0004087
	$f(K_i)$	0.0003808	0.0003513	0.0003815	0.0003503	0.0003512	0.0003225
	Ranked:	2 nd	3 rd	1st	5 th	4 th	6 th
IV	K_i^+	0.0007047	0.0006739	0.0007047	0.0006697	0.0006733	0.0005911
	K_i^-	0.0004708	0.0004502	0.0004708	0.0004474	0.0004498	0.0003949
	$f(K_i)$	0.0003714	0.0003552	0.0003714	0.0003530	0.0003549	0.0003116
	Ranked:	1st	3 rd	2 nd	5 th	4 th	6 th

ative ideal solution (D_i^-) and the relative closeness of each alternative to the ideal solution (C_i), with $i \in \{1, \dots, m\}$. The values of these three measures and the rankings are presented for each preference approach in Table 13.

The rankings produced by the TOPSIS method show clear discrepancies with those obtained from the other four methods. Specifically, across all four preference approaches, TOPSIS ranks alternative

Table 12

Final results of the COPRAS method for all preference approaches

Preference Approach	Final results	A1	A2	A3	A4	A5	A6
I	S_{+i}	0.02024	0.02167	0.01989	0.02145	0.02165	0.02010
	S_{-i}	0.13485	0.15018	0.13417	0.14776	0.14979	0.15825
	Q_i	0.17739	0.16278	0.17783	0.16487	0.16312	0.15401
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
II	S_{+i}	0.01322	0.01415	0.01299	0.01401	0.01414	0.01313
	S_{-i}	0.13972	0.15737	0.13899	0.15543	0.15707	0.16978
	Q_i	0.18006	0.16228	0.18071	0.16398	0.16255	0.15043
	Ranked:	2 nd	5 th	1st	3 rd	4 th	6 th
III	S_{+i}	0.02858	0.03060	0.02807	0.03028	0.03056	0.02838
	S_{-i}	0.12398	0.14114	0.12367	0.13964	0.14089	0.15421
	Q_i	0.17961	0.16327	0.17948	0.16437	0.16346	0.14980
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th
IV	S_{+i}	0.01963	0.02102	0.01928	0.02080	0.02099	0.01949
	S_{-i}	0.13696	0.14573	0.13712	0.14530	0.14564	0.16804
	Q_i	0.17553	0.16754	0.17501	0.16775	0.16761	0.14656
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th

Table 13

Final results of the TOPSIS method for all preference approaches

Preference Approach	Final results	A1	A2	A3	A4	A5	A6
I	D_i^+	0.02446	0.04077	0.02564	0.03374	0.03967	0.04211
	D_i^-	0.04900	0.03218	0.05082	0.03205	0.03207	0.03559
	C_i	0.66703	0.44113	0.66467	0.48714	0.44699	0.45805
	Ranked:	1st	6 th	2 nd	3 rd	5 th	4 th
II	D_i^+	0.01614	0.03568	0.01679	0.03045	0.03485	0.05022
	D_i^-	0.05536	0.03449	0.05640	0.03455	0.03444	0.02906
	C_i	0.77422	0.49156	0.77054	0.53159	0.49705	0.36653
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th
III	D_i^+	0.01779	0.03383	0.01888	0.03013	0.03321	0.04777
	D_i^-	0.05073	0.03117	0.05129	0.03073	0.03107	0.02513
	C_i	0.74036	0.47955	0.73089	0.50495	0.48333	0.34472
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th
IV	D_i^+	0.02928	0.02740	0.03087	0.02577	0.02708	0.05423
	D_i^-	0.04948	0.04102	0.04930	0.03978	0.04087	0.01756
	C_i	0.62820	0.59954	0.61491	0.60690	0.60142	0.24455
	Ranked:	1st	5 th	2 nd	3 rd	4 th	6 th

A1 in the top position and alternative A3 as the second-best choice. Concurrently, except in the first preference approach, the remaining alternatives mirror the positioning found in the PIV, PARIS and COPRAS methods.

3.3 Comparative Analysis and Rank Correlation of Alternative Rankings

The comparative analysis in this subsection is carried out to evaluate the consistency of the rankings produced by different MCDM approaches and to examine the robustness of the obtained results with respect to the selection of the decision-making method. The level of agreement among the applied MCDM methods is used to evaluate the reliability of the obtained rankings.

To this end, Tables 14–17 summarize the rankings from Tables 9–12 for each preference approach considered in this study.

Table 14

Alternative rankings obtained using the PIV, PARIS, MARCOS and COPRAS methods for Preference Approach I

Alternative	Rankings			
	PIV	PARIS	MARCOS	COPRAS
A1	2	2	2	2
A2	5	5	3	5
A3	1	1	1	1
A4	3	3	5	3
A5	4	4	4	4
A6	6	6	6	6

Table 15

Alternative rankings obtained using the PIV, PARIS, MARCOS and COPRAS methods for Preference Approach II

Alternative	Rankings			
	PIV	PARIS	MARCOS	COPRAS
A1	2	2	2	2
A2	5	5	3	5
A3	1	1	1	1
A4	3	3	5	3
A5	4	4	4	4
A6	6	6	6	6

Table 16

Alternative rankings obtained using the PIV, PARIS, MARCOS and COPRAS methods for Preference Approach III

Alternative	Rankings			
	PIV	PARIS	MARCOS	COPRAS
A1	1	1	2	1
A2	5	5	3	5
A3	2	2	1	2
A4	3	3	5	3
A5	4	4	4	4
A6	6	6	6	6

As can be seen from the data presented in Tables 14, 15, 16 and 17, when each preference approach is considered separately, the PIV, PARIS and COPRAS methods produced identical alternative rankings, whereas the MARCOS method yielded a somewhat different ranking. Nevertheless, except

Table 17

Alternative rankings obtained using the PIV, PARIS, MARCOS and COPRAS methods for Preference Approach IV

Alternative	Rankings			
	PIV	PARIS	MARCOS	COPRAS
A1	1	1	1	1
A2	5	5	3	5
A3	2	2	2	2
A4	3	3	5	3
A5	4	4	4	4
A6	6	6	6	6

for Preference Approach III, it also identified the same alternative as the best solution as the other three methods. Furthermore, it should be noted that, across all four preference approaches, all four methods consistently ranked alternatives A5 and A6 in fourth and sixth place, respectively.

For comparison with the results obtained using the TOPSIS method, which is adopted as a reference method in this study, Figures 2, 3, 4 and 5 present, for each preference approach, comparisons of the rankings obtained using each of the four examined methods individually with the ranking obtained by applying the TOPSIS method.

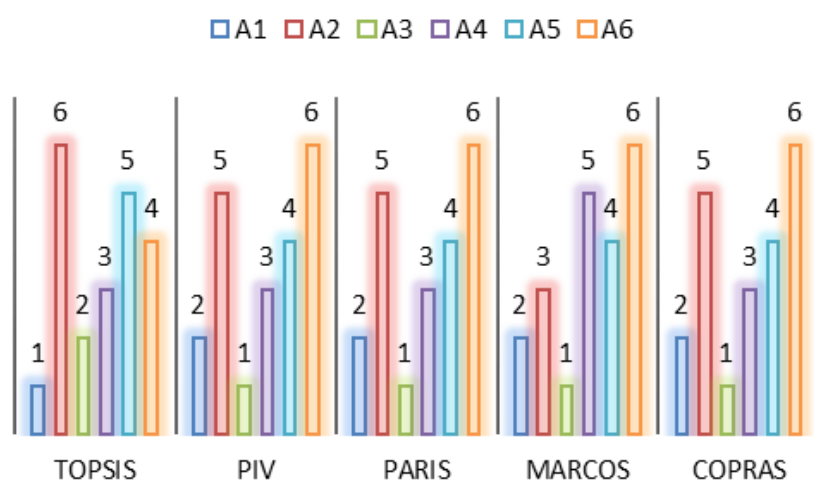


Fig. 2. Comparison of the rankings obtained using the considered MCDM methods with the TOPSIS ranking (Preference Approach I)

An inspection of Figures 2–5 suggests the need to examine the correlation between the rankings obtained in this study. This analysis is therefore carried out in what follows.

The values of the Spearman rank correlation coefficient calculated based on the rankings determined using the PIV, PARIS, MARCOS and COPRAS methods, compared to those obtained using the TOPSIS method and the significance of correlation (for each preference approach in this study), are shown in Table 18. The threshold, i.e. the critical value based on which conclusions were drawn about the significance of the correlation, was 0.886 (see Subsection 2.5).

Since all rankings obtained using the MARCOS method differ from the corresponding ones obtained applying all the remaining methods, it is worth showing data representing correlation of alternative rankings when comparing the MARCOS method with the PIV, PARIS and COPRAS methods. These data are included in Table 19.

The data in Table 18 show that the correlation between the ranking lists obtained using the PIV,

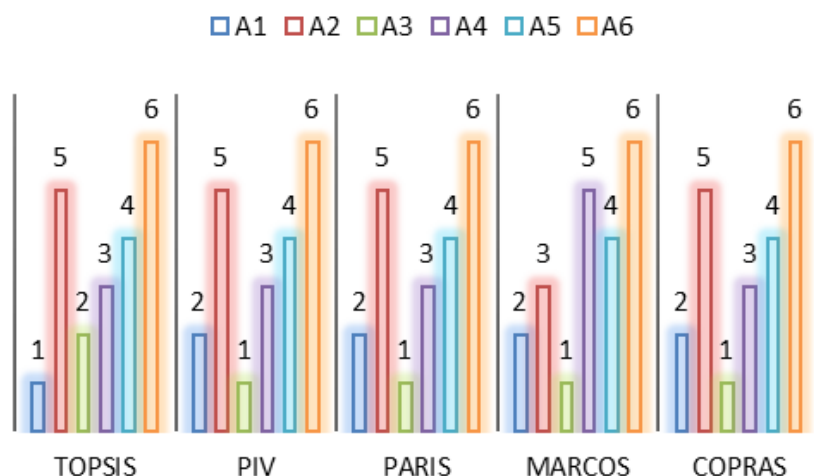


Fig. 3. Comparison of the rankings obtained using the considered MCDM methods with the TOPSIS ranking (Preference Approach II)

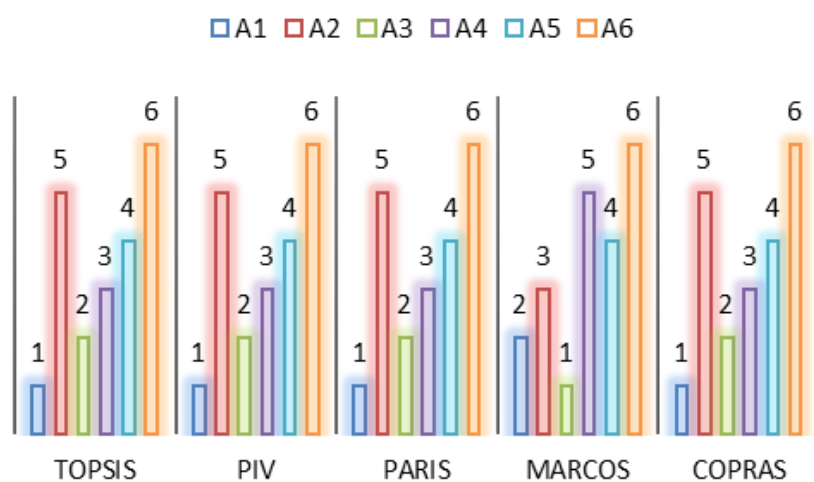


Fig. 4. Comparison of the rankings obtained using the considered MCDM methods with the TOPSIS ranking (Preference Approach III)

Table 18

Spearman rank correlation coefficients between the PIV, PARIS, MARCOS and COPRAS methods, and the TOPSIS method (all preference approaches)

Preference Approach	Spearman rank correlation coefficient				
		PIV	PARIS	MARCOS	COPRAS
I	$S_{\mu, \text{TOPSIS}}$	0.771	0.771	0.429	0.771
	Correlation:	insignificant	insignificant	insignificant	insignificant
II	$S_{\mu, \text{TOPSIS}}$	0.943	0.943	0.714	0.943
	Correlation:	significant	significant	insignificant	significant
III	$S_{\mu, \text{TOPSIS}}$	1.000	1.000	0.714	1.000
	Correlation:	perfect	perfect	insignificant	perfect
IV	$S_{\mu, \text{TOPSIS}}$	1.000	1.000	0.771	1.000
	Correlation:	perfect	perfect	insignificant	perfect

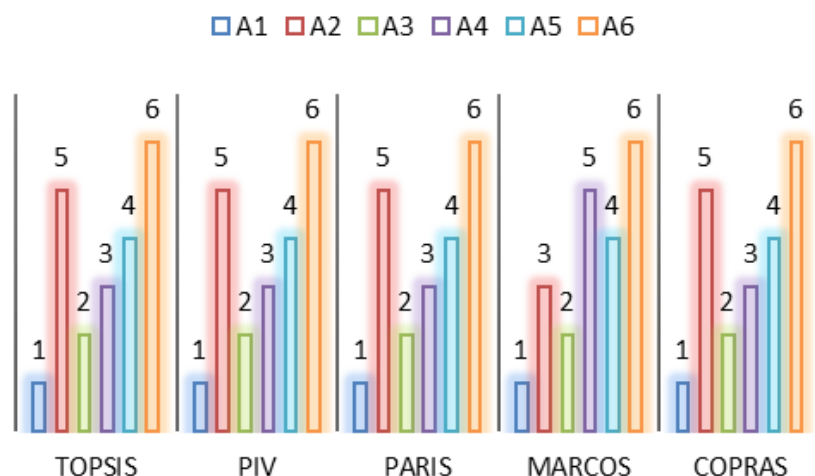


Fig. 5. Comparison of the rankings obtained using the considered MCDM methods with the TOPSIS ranking (Preference Approach IV)

Table 19

Spearman rank correlation coefficients between the MARCOS method and the PIV, PARIS and COPRAS methods (all preference approaches)

Preference Approach		Spearman rank correlation coefficient		
		$S_{\text{MARCOS-PIV}}$	$S_{\text{MARCOS-PARIS}}$	$S_{\text{MARCOS-COPRAS}}$
I		0.771	0.771	0.771
	Correlation:	insignificant	insignificant	insignificant
II		0.771	0.771	0.771
	Correlation:	insignificant	insignificant	insignificant
III		0.714	0.714	0.714
	Correlation:	insignificant	insignificant	insignificant
IV		0.771	0.771	0.771
	Correlation:	insignificant	insignificant	insignificant

PARIS and COPRAS methods and the corresponding reference ranking list is perfect in the third and fourth preference approaches, significant (very strong) in the second, whereas a negligible correlation of this type exists in the first preference approach (see Table 18). The exact opposite is true for the MARCOS method. Namely, its rankings exhibit a negligible correlation with the corresponding lists of the reference method, as well as those of the PIV, PARIS and COPRAS methods (see Table 19).

3.4 Sensitivity Analysis

To further validate the obtained results, a sensitivity analysis was conducted using the leave-one-out approach. In each scenario, one evaluation criterion was excluded from the decision model, while the weights of the remaining criteria were renormalized to ensure that their total sum remained equal to one. Specifically, Scenario 1 (Sc-1) corresponds to the exclusion of criterion F1, Scenario 2 (Sc-2) to the exclusion of F2, and so forth, until Scenario 8 (Sc-8), which corresponds to the exclusion of F8.

The alternatives were subsequently re-evaluated and re-ranked using the PIV, PARIS, MARCOS and COPRAS methods. The resulting rankings were then compared with those obtained for the baseline model in order to assess the stability of the ranking results under successive criterion exclusion. The observed ranking changes provide insight into the influence of individual criteria on the final decision and enable an assessment of the stability and reliability of the proposed decision-making framework.

The ranking positions of the alternatives obtained for the baseline model and all leave-one-out

sensitivity analysis scenarios are presented in Figures 6–9 for Preference Approaches I–IV, respectively.

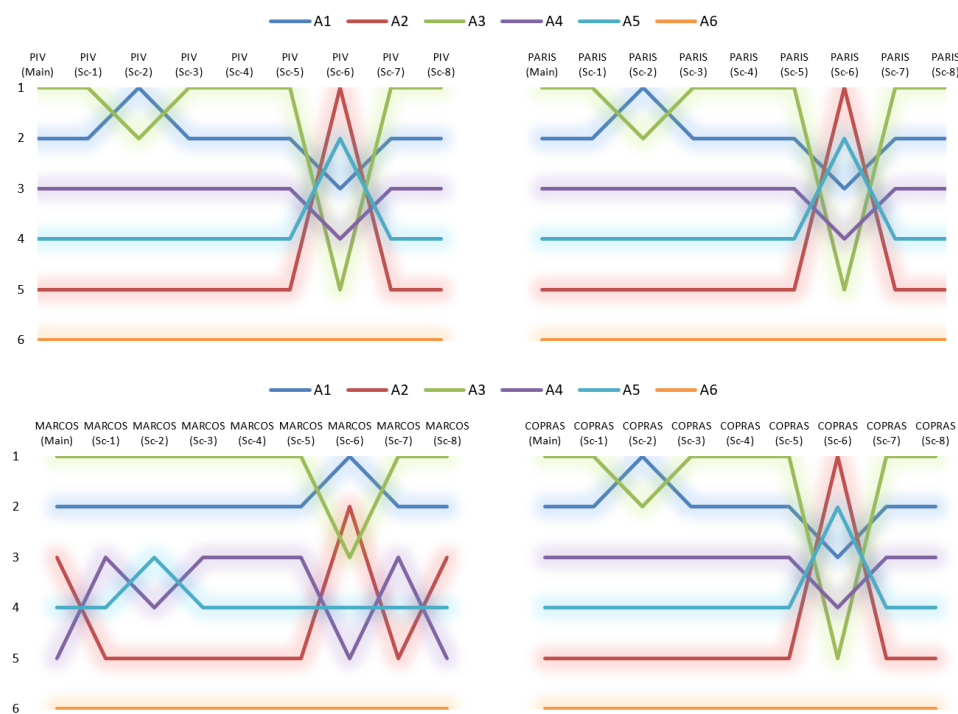


Fig. 6. Alternative ranking positions for the baseline (main) ranking and the leave-one-out sensitivity analysis scenarios (Sc-1 to Sc-8) under Preference Approach I

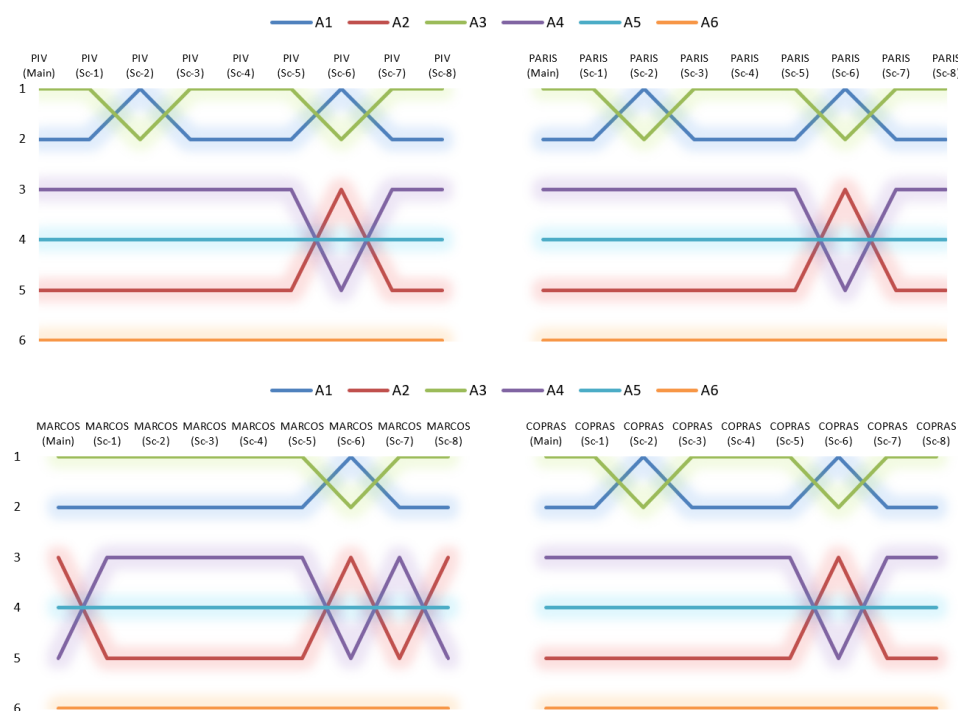


Fig. 7. Alternative ranking positions for the baseline (main) ranking and the leave-one-out sensitivity analysis scenarios (Sc-1 to Sc-8) under Preference Approach II

The sensitivity analysis under Preference Approach I (Figure 6) demonstrates that the exclusion of individual criteria does not have an equal impact on the final rankings. Criterion F6 exerts the strongest influence on the ranking results, whereas the exclusion of the remaining criteria generally leads either to no changes or only limited local rearrangements in the ranking of alternatives. The

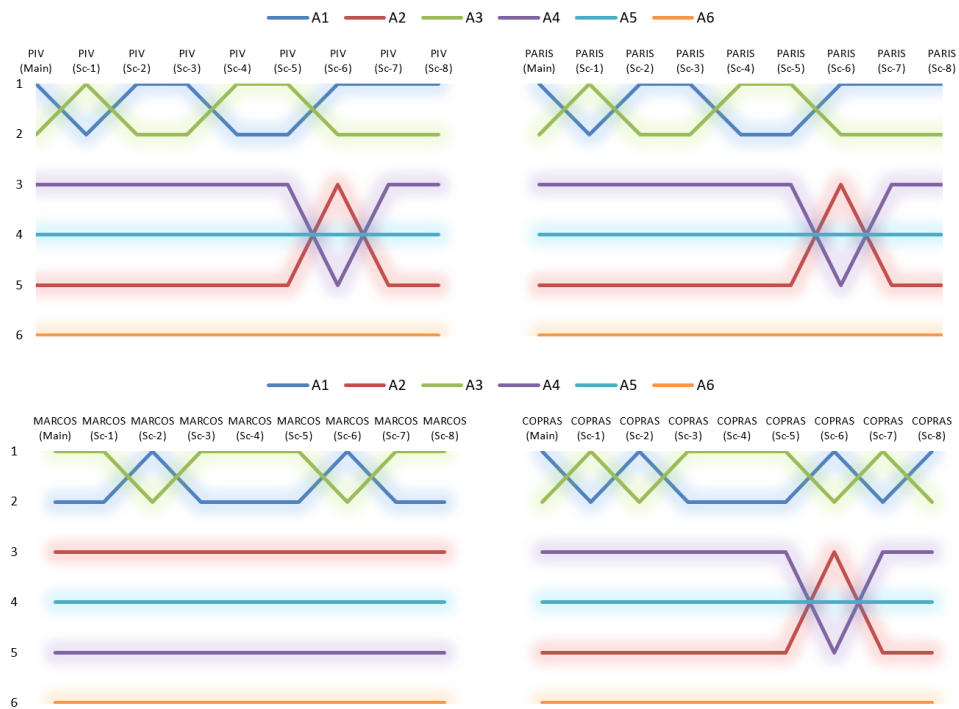


Fig. 8. Alternative ranking positions for the baseline (main) ranking and the leave-one-out sensitivity analysis scenarios (Sc-1 to Sc-8) under Preference Approach III

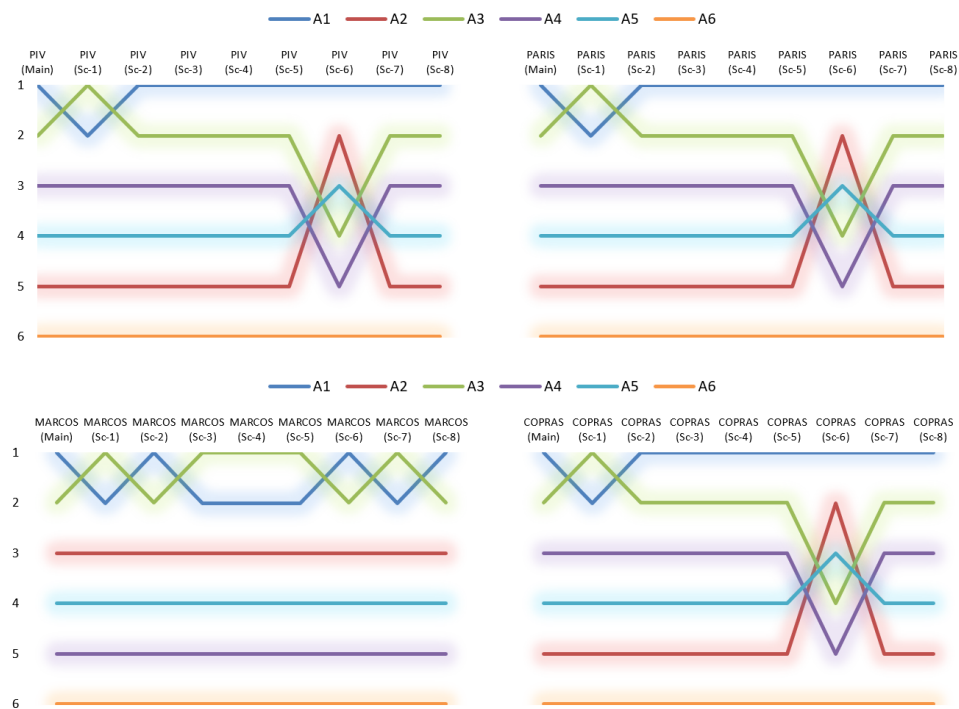


Fig. 9. Alternative ranking positions for the baseline (main) ranking and the leave-one-out sensitivity analysis scenarios (Sc-1 to Sc-8) under Preference Approach IV

PIV, PARIS and COPRAS methods exhibit completely identical response patterns throughout the entire analysis, while MARCOS follows a different ranking adjustment pattern. Nevertheless, the observed rank variations do not affect the identification of the best- and worst-ranked alternatives, indicating that the final decision remains stable despite the exclusion of individual criteria.

Regarding the sensitivity analysis under Preference Approach II (Figure 7), the ranking behaviour indicates that the influence of criterion exclusion also varies among the analysed scenarios. Most ranking changes are confined to alternatives occupying intermediate positions in the ranking list, whereas the best- and worst-ranked alternatives preserve their positions. The PIV, PARIS and COPRAS methods again exhibit completely identical response patterns, while MARCOS follows a different ranking adjustment mechanism. These findings indicate that criterion exclusion affects the relative ordering of neighbouring alternatives but does not alter the overall decision outcome.

Under Preference Approach III (Figure 8), the obtained ranking results show that the response of the applied MCDM methods depends on both the excluded criterion and the adopted weighting structure. In this case, the PIV and PARIS methods continue to exhibit identical local ranking changes, whereas the sensitivity of the COPRAS method slightly differs with respect to the first two positions in the ranking list; however, it still identifies the same two best-ranked alternatives as the PIV and PARIS methods. MARCOS becomes considerably less sensitive (and even completely insensitive for positions three to six), producing only minor variations throughout the analysed scenarios. This behaviour indicates that the response of the MARCOS method is primarily governed by the adopted preference structure rather than by criterion exclusion itself. Despite these differences, the obtained rankings also remain sufficiently stable to preserve the final decision outcome.

The ranking patterns obtained under Preference Approach IV (Figure 9) further illustrate the influence of the adopted weighting structure on the response of the analysed MCDM methods. The PIV, PARIS and COPRAS methods again exhibit completely identical ranking patterns, whereas MARCOS shows only limited ranking changes, similar to those observed under Preference Approach III, which are confined only to the first two positions in the ranking list. Although several local changes occur among the intermediate-ranked alternatives (for the PIV, PARIS and COPRAS methods), with a behaviour somewhat comparable to that observed under Preference Approach I, the exclusion of individual criteria does not affect the identification of the most and least preferable solutions. These results provide additional evidence that the final decision remains stable despite local ranking variations.

4. Conclusions

This study proposed a decision-making framework for determining the optimal measurement accuracy in geodetic control networks intended for monitoring engineering structure displacements. The framework integrates the SWING weighting method with four MCDM methods, namely PIV, PARIS, MARCOS and COPRAS, and evaluates six competitive alternatives under four different preference approaches. In addition to comparing the obtained ranking results, the proposed framework incorporates a leave-one-out sensitivity analysis to assess the stability and consistency of the decision-making process under successive criterion exclusion scenarios.

The obtained results demonstrate that all four MCDM methods are suitable for the considered optimization problem. Although some differences in ranking behaviour were observed, particularly for MARCOS, all methods consistently identified alternatives A1 and A3 as the most favourable solutions. Their mutual positions varied depending on the adopted preference approach, indicating that both alternatives provide highly competitive solutions with very similar measurement characteristics. Since the principal difference between alternatives A1 and A3 lies in the accuracy of distance measurements between the reference points, either of the two total stations corresponding to alternatives A1 and A3 may be considered an appropriate choice for the considered monitoring task.

The comparative analysis and the performed sensitivity analysis provided additional insight into the behaviour of the analysed MCDM methods. For each individual preference approach, the PIV and PARIS methods exhibited fully identical ranking patterns throughout all analysed sensitivity scenarios. The COPRAS method exhibited almost the same ranking behaviour as PIV and PARIS, differing only under Preference Approach III, where more frequent exchanges between the two highest-ranked alternatives were observed. In contrast, MARCOS exhibited greater variability across the four examined preference approaches, suggesting that its ranking behaviour is more sensitive to changes in the adopted preference structure. More importantly, the obtained results indicate that, for the considered optimization problem, the stability of the rankings is influenced more strongly by the adopted preference structure than by the particular MCDM method used to generate them. This finding highlights the importance of carefully defining criterion weights, as the adopted weighting structure may exert a greater influence on the final decision than the selection of the ranking algorithm itself.

The contribution of this study is twofold. From an application perspective, the study introduces the PIV, PARIS, MARCOS and COPRAS methods into the optimization of measurement accuracy for engineering structure displacement monitoring, an engineering geodesy problem that has not previously been addressed within the MCDM framework. Unlike previous studies, which primarily focused on optimizing network configuration or observation plans, the present research addresses the selection of measurement accuracy while maintaining a fixed network geometry and observation plan. From a methodological perspective, the study demonstrates that combining multiple preference structures with comparative analysis and sensitivity analysis provides a more comprehensive evaluation of MCDM behaviour than assessments performed under a single weighting scenario. This integrated approach enables a more reliable interpretation of ranking consistency and stability, thereby strengthening confidence in the obtained decision outcomes.

The proposed framework also has important practical implications. It provides geodetic agencies and engineering companies with a transparent, systematic and reproducible procedure for selecting measurement equipment according to predefined performance criteria and decision-maker preferences. Beyond identifying the most appropriate alternative, the framework offers a rational basis for justifying the selected measurement accuracy, thereby reducing subjectivity and increasing confidence in engineering decision-making processes related to displacement monitoring.

The present study is subject to certain limitations. The investigated case study was based on a relatively small, atypical geodetic micro-network, with line lengths ranging from 9.2 m to 197.4 m, while the observation plan remained unchanged for all analysed alternatives. Consequently, the reported findings should be interpreted within the context of the considered optimization problem. Future research should examine the applicability of the proposed framework to geodetic control networks of larger engineering structures, such as dams, long-span bridges, or other large-scale monitoring networks characterized by substantially longer observation distances. In addition, future studies should evaluate other MCDM methods, weighting techniques and sensitivity analysis strategies in order to further assess the generality of the proposed decision-making framework.

Beyond the specific engineering geodesy application considered in this study, the obtained results have broader methodological implications for comparative MCDM research. They indicate that meaningful evaluation of ranking methods requires not only the comparison of different decision-making algorithms, but also the consideration of multiple preference structures supported by complementary sensitivity analyses. For the considered optimization problem, the findings further suggest that the adopted preference structure exerts a greater influence on ranking stability than the particular MCDM method itself. Consequently, comparative studies based on a single weighting scenario may provide only a partial understanding of MCDM behaviour under varying decision-maker preferences. Incorporating multiple preference structures into comparative analyses therefore enables a more comprehensive assessment of ranking behaviour, increases confidence in the obtained decision outcomes,

and establishes a stronger methodological foundation for future MCDM applications in engineering geodesy and related engineering disciplines.

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Conflicts of Interest

The authors declare no conflicts of interest.

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